

Long-Run Risk through Consumption Smoothing

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Abstract

We show that a standard production economy model where consumers have Epstein-Zin preferences can jointly explain the low volatility of consumption growth and a high market price of risk with a low coefficient of relative risk aversion (five). *Endogenous* consumption smoothing increases the price of risk in this economy as it induces highly persistent time-variation in expected aggregate consumption growth (long-run risk), even when technology growth is i.i.d. The model identifies an observable proxy for otherwise hard to measure expected consumption growth. Using this proxy, we test and find support for key predictions of the model in the time series of consumption growth and the cross-section of stock returns.

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1 Introduction

Long-run consumption risk has recently been proposed as a mechanism for explaining important asset price moments such as the Sharpe ratio of equity market returns, the equity premium, the level and volatility of the risk free rate and the cross-section of stock returns (see Bansal and Yaron, 2004, Hansen, Heaton and Li, 2005, and Parker and Julliard, 2005). In this paper, we investigate how long-run consumption risk arises *endogenously* in a standard production economy framework and how this additional risk factor can help production economy models to jointly explain the dynamic behavior of consumption, investment and asset prices.¹

We deviate from the standard production economy model by assuming that consumers have Epstein-Zin preferences.² Unlike in the power utility case, where risk is only associated with the shock to *realized* consumption growth, investors with Epstein-Zin preferences also demand a premium for holding assets correlated with shocks to *expected* consumption growth. The latter source of risk has been labelled "long-run risk" in previous literature (Bansal and Yaron, 2004). In production economy models, endogenous long-run risk arises because consumption smoothing induces highly persistent time-variation in expected consumption growth rates. We show that this endogenous long-run risk can substantially increase the price of risk in the economy. The production economy model with Epstein-Zin preferences can then explain the high Sharpe ratio of equity returns with a low volatility of consumption growth and a low coefficient of relative risk aversion.

Why does the consumer optimally choose a consumption process that leads to a high price of risk? The price of risk is related to risk across states, while the agent maximizes the level of expected utility which also is a function of substitution across time. The agent thus trades off the benefit of shifting consumption across time with the cost of higher volatility of marginal utility across states. Asset prices in the production economy reflect the optimal outcome of this trade-off.

In equilibrium, time-varying expected consumption growth turns out to be a small, but

¹For extensive discussions of the poor performance of standard production economy models in terms of jointly explaining asset prices and macroeconomic moments, refer to Rouwenhorst (1995), Lettau and Uhlig (2000), Uhlig (2004), and Cochrane (2005), amongst others.

²Epstein-Zin preferences provide a convenient separation of the elasticity of intertemporal substitution (ψ) from the coefficient of relative risk aversion (γ), which are forced to $\gamma = \frac{1}{\psi}$ in the power utility case. If $\gamma > \frac{1}{\psi}$, investors prefer early resolution of uncertainty and are averse to time-varying expected consumption growth. If $\gamma < \frac{1}{\psi}$, investors prefer late resolution of uncertainty and *like* shocks to expected consumption growth.

highly persistent, fraction of realized consumption growth. When the model is calibrated to fit standard macroeconomic moments, the *endogenous* expected consumption growth rate process is quantitatively very close to the exogenous processes that have been specified in the recent asset pricing literature (see, e.g., Bansal and Yaron, 2004). Note that this result is of particular interest since it is very difficult to empirically distinguish a small predictable component of consumption growth from i.i.d. consumption growth given the short sample of data we have available (see Harvey and Shepard, 1990, and Hansen, Heaton and Li, 2005, amongst others). Bansal and Yaron (2004), for instance, *calibrate* a process for consumption growth with a highly persistent trend component and demonstrate that their process can match a number of moments of aggregate consumption growth. In lieu of robust empirical evidence on this matter, the model presented in this paper provides a theoretical justification for long-run risk dynamics in aggregate consumption growth arising from optimal consumption smoothing.

The persistence of the technology shocks is crucial for the asset pricing implications of long-run risk in the model. Consider the case when agents have a preference for early resolution of uncertainty (the relative risk aversion is less than the reciprocal of the elasticity of intertemporal substitution). Then, permanent technology shocks lead to time-varying expected consumption growth that increases the price of risk in the economy, while transitory technology shocks lead to time-varying expected consumption growth that decreases the price of risk. The intuition for this is as follows. A permanent positive shock to productivity implies a permanently higher optimal level of capital. As a result, investors increase investment in order to build up a higher capital stock. High investment today implies low current consumption, but high future consumption. Thus, expected consumption growth is high. Since agents in this economy dislike negative shocks to future economic growth prospects, both shocks to expected consumption growth and realized consumption growth are risk factors. Furthermore, the shocks are positively correlated and thus reinforce each other. Therefore, endogenous consumption smoothing increases the price of risk in the economy if agents have a preference for early resolution of uncertainty and technology shocks are permanent. If, on the other hand, shocks to technology are transitory, the endogenous long-run risk *decreases* the price of risk in the economy. A transitory, positive shock to technology implies that technology is expected to revert back to its long-run trend. Thus, if realized consumption growth is high, expected future long-run consumption growth is low as consumption also reverts to the long-run trend. The shock to expected future consumption growth is now *negatively* correlated with the shock to realized consumption growth, and the

long-run risk component acts as a hedge for shocks to realized consumption.³ The overall price of risk in the economy is then decreasing in the magnitude of long-run risk. In the case when agents have a preference for *late* resolution of uncertainty - i.e., the elasticity of intertemporal substitution is less than the reciprocal of the coefficient of relative risk aversion - agents *like* long-run risk. Now endogenous long-run risk increases the price of risk when technology shocks are transitory and decrease the price of risk when technology shocks are permanent.

We evaluate the quantitative effects of transitory and permanent technology shocks on aggregate macroeconomic and financial moments with calibrated versions of our model. We identify two particularly interesting cases. First, we show that a model with *transitory* technology shocks and a *low* elasticity of intertemporal substitution can jointly explain the low volatility of consumption growth and the high price of risk with a low coefficient of relative risk aversion. This model has a high equity return volatility and risk premium, as in the data. However, the model generates too high volatility in the risk free rate. Second, we show that a model with *permanent* technology shocks and a relatively *high* elasticity of intertemporal substitution also can jointly explain the low volatility of consumption growth and the high price of risk with a low coefficient of relative risk aversion. Furthermore, this model has low volatility in the risk free rate, as in the data. The equity premium, however, is too low in this model. We show that this is due to low capital adjustment costs and counter-cyclical dividends. We address this problem by calibrating the wage process of the model to the data. This brings the endogenous dividend process closer to the data, and as a result the equity premium as well as the equity return volatility increase by an order of magnitude to levels close to empirical values. We conclude from this calibration that the standard real business cycle model (without habit preferences) has the clear potential to jointly explain asset prices and macroeconomic time series, without unrealistic levels of risk aversion and excessive risk free rate volatility.

The production economy model relates the aggregate level of technology (total factor productivity), consumption, and investment to the dynamic behavior of aggregate consumption growth. We use this link to derive new testable implications. Our model implies that

³This description is intentionally loose to emphasize the intuition. The consumption response to transitory technology shocks is often hump-shaped. Thus, a positive shock to realized consumption growth is followed by high expected consumption growth in the near term, but lower expected consumption growth in the long term - the negative correlation arises at lower frequencies. The low frequency effect dominates for standard values of the discount factor and leads to a lower price of risk unless the transitory shocks are extremely persistent.

the ratio of total factor productivity to consumption is a good proxy for the otherwise hard to measure expected consumption growth rate. We find empirical support for the permanent shock model by showing that the ratio of log total factor productivity to consumption forecasts future consumption growth over long horizons with a positive sign. We furthermore test a linear approximation of the model on the cross-section of stock returns and show, using the above proxy, that shocks to *expected* consumption growth are a priced risk factor that substantially improves the ability of the Consumption CAPM to explain the cross-section of stock returns.

We proceed as follows. We start by providing an overview of related literature. Then we develop and interpret the model. In Section 4 we calibrate and solve the model, demonstrate and interpret results, and provide intuition. In Section 5 we test some empirical implications of our model. Section 6 concludes.

2 Related Literature

This paper is mainly related to three strands of the literature: the literature on consumption smoothing, the literature on long-run risk, and the literature that aims to jointly explain macroeconomic aggregates and asset prices.

It is well-known that (risk averse) agents want to smooth consumption over time. The permanent income hypothesis of Friedman (1957) is the classic reference. Hall (1978) is a seminal empirical investigation of this hypothesis. Hall shows that consumption should approximately follow a random walk and finds support for this in the data. The results in our paper are consistent with Hall: We also find that consumption should be very close to a random walk. But, different from Hall, we emphasize that consumption growth has a small, highly persistent, time-varying component. Time-variation in expected growth rates, arising from consumption smoothing in production economy models, has also been pointed out before. For example, Den Haan (1995) demonstrates that the risk free rate in production economy models is highly persistent (close to a random walk) even when the level of technology is i.i.d. Campbell (1994) solves a log-linear approximation to the standard real business cycle model with power utility preferences, which provides an analytical account of how the optimal consumption-savings decisions lead to time-varying expected consumption growth in this model.

Bansal and Yaron (2004) show that a small, persistent component of consumption growth

can have quantitatively important implications for asset prices if the representative agent has Epstein-Zin preferences. Bansal and Yaron term this source of risk "long-run risk" and show that it can explain many aspects of asset prices. They specify exogenous processes for dividends and consumption with a slow-moving expected growth rate component and demonstrate that the ensuing long-run consumption risk greatly improves their model's performance with respect to asset prices without having to rely on, e.g., habit formation and the high relative risk aversion such preferences imply. We show that the process for consumption Bansal and Yaron assume as exogenous can be generated endogenously in a standard production economy model with Epstein-Zin preferences and the same preference parameters Bansal and Yaron use. Since it is very difficult to empirically distinguish between i.i.d. consumption growth and consumption growth with a very small, highly persistent time-varying component, this result is of particular importance for the Bansal and Yaron framework. Hansen, Heaton and Li (2005) emphasize this point in their study of the impact of long-run risk on the cross-section of stock returns. We also consider the implications for aggregate investment, which Bansal and Yaron abstract from, and the aggregate dividend process in our model is endogenous.

A recent paper that generates interesting consumption dynamics is due to Panageas and Yu (2006). These authors focus on the impact of major technological innovations and real options on consumption and the cross-section of asset prices. They assume, as do we, the technology process to be i.i.d. The major technological innovations, however, are assumed to occur at a very low frequency (about 20 years), and are shown to carry over into a small, highly persistent component of aggregate consumption. In that sense, Panageas and Yu assume, contrary to us, the frequency of the predictable component of consumption growth. Moreover, time-variation in expected consumption growth (long-run risk) is not itself a priced risk factor in the Panageas and Yu model because the representative agent does not have Epstein-Zin preferences, but external ratio-habit as in Abel (1990). Panageas and Yu require that investment is irreversible, whereas we allow for a convex adjustment cost function. Also, since investment in their model means paying a "gardener" to plant a tree, their model does not have a clear separation of investment and labor income.

Parker and Julliard (2005) find that the Consumption CAPM can empirically explain a large fraction of the cross-sectional dispersion in average excess stock returns only when consumption growth is measured over longer horizons. This is consistent with the presence of long-run risks. Bansal, Kiku and Yaron (2006) explicitly test and find considerable support for the long-run risk model in the cross-section of stock returns.

There are quite a few papers before Bansal and Yaron (2004) that emphasize a small, highly persistent component in the pricing kernel. An early example is Backus and Zin (1994) who use the yield curve to reverse-engineer the stochastic discount factor and find that it has high conditional volatility and a persistent, time-varying conditional mean with very low volatility. These dynamics are also highlighted in Cochrane and Hansen (1992). This is exactly the dynamic behavior generated endogenously by the models considered in this paper, and as such the paper complements the above earlier studies. The use of Epstein-Zin preferences provides a justification for why the small, slow-moving time-variation in expected consumption growth generates high volatility of the stochastic discount factor. These preferences have become increasingly popular in the asset pricing literature. By providing a convenient separation between the coefficient of relative risk aversion and the elasticity of intertemporal substitution, they help to jointly explain asset market data and aggregate consumption dynamics. An early implementation is Epstein and Zin (1991), while Malloy, Moskowitz and Vissing-Jorgensen (2005) and Yogo (2006) are more recent, successful examples.

This paper also makes a contribution to a literature Cochrane (2005) terms ‘production-based asset pricing’. This literature tries to jointly explain the behavior of macroeconomic time series, in particular aggregate consumption, and asset prices. The starting point of this literature is the standard production economy model (standard stochastic growth model) with capital adjustment costs and the observation that this model, while being able to generate realistic processes for consumption and investment, fails markedly at explaining asset prices.⁴

Both Jermann (1998) and Boldrin, Christiano, Fisher (2001) augment the basic production economy framework with habit preferences in order to remedy its shortcomings. Boldrin, Christiano, Fisher also assume a two-sector economy with adjustment frictions across sectors and across time. Boldrin, Christiano, Fisher furthermore endogenize the labor-leisure decision, they assume however that labor can not be adjusted immediately in response to technology shocks. Jermann, and in particular Boldrin, Christiano, Fisher, succeed to a considerable extent to jointly explain with their models macroeconomic time series and asset prices. However, the price both models pay, typical for simple internal habit specifications, is excessive volatility of the risk free rate and very high levels of risk aversion. In a sense, their internal habits buy volatility in equity returns with volatility in risk free rates, which is

⁴Cochrane (2005): "[Jermann (1998)] starts with a standard real business cycle (one sector stochastic growth) model and verifies that its asset-pricing implications are a disaster."

counter-factual. This is also problematic, because a too volatile risk free rate implies that the conditional mean of the stochastic discount factor is too volatile. Relative to Jermann and Boldrin, Christiano, Fisher our contribution is to demonstrate that the standard production economy model without habit preferences can actually, once appropriately calibrated, jointly explain basic macroeconomic time series as well as important aggregate asset price moments *without* excessive risk free rate volatility and high levels of risk aversion.

Tallarini (2000) proposes a model that is closely related to our setup. Tallarini restricts himself to a special case of our model with the elasticity of intertemporal substitution fixed at unity and no capital adjustment costs. By increasing the coefficient of relative risk aversion to very high levels, Tallarini manages to match some asset pricing moments such as the market price of risk (Sharpe ratio) as well as the level of the risk free rate, while equity premium and return volatilities in his model remain basically zero. We differ from Tallarini in that our focus is on changing the elasticity of intertemporal substitution and the implications for the existence and pricing of long-run risk. Relative to the Tallarini setup we show that (moderate) capital adjustment costs together with an elasticity of intertemporal substitution different from unity can dramatically improve the model's ability to match asset pricing moments. We confirm Tallarini's conclusion that the behavior of macroeconomic time series is driven by the elasticity of intertemporal substitution and largely unaffected by the coefficient of relative risk aversion. However, we do not confirm a "separation theorem" of quantity and price dynamics. When we change the elasticity of intertemporal substitution in our model, both macroeconomic quantity and asset price dynamics are greatly affected.

In recent research, Croce (2007) investigates the welfare implications of long-run risk in a general equilibrium production economy similar to the one we analyze. Finally, Campanale, Castro, and Clementi (2007) look at asset prices in general equilibrium production economies where the representative agent's preferences are in the Chew-Dekel class. Contrary to us, they do not consider the role of long-run risk.

3 The Model

The model is a standard real business cycle model (Kydland and Prescott, 1982, and Long and Plosser, 1983). There is a representative firm with Cobb-Douglas production technology and capital adjustment costs, and a representative agent with Epstein-Zin preferences. Our objective is to demonstrate how standard production economy models endogenously give rise to long-run consumption risk and that this long-run risk can improve the performance of

these models in explaining important moments of asset prices. To that end we keep both production technology as well as the process for total factor productivity as simple and as standard as possible. We describe the key components of our model in turn.

The Representative Agent. We assume a representative household whose preferences are in the recursive utility class of Epstein and Zin (1989):

$$U_t(C_t) = \left\{ (1 - \beta) C_t^{\frac{1-\gamma}{\theta}} + \beta (E_t [U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \quad (1)$$

where E_t denotes the expectation operator, C_t denotes aggregate consumption, β the discount factor, and $\theta = \frac{1-\gamma}{1-1/\psi}$. Epstein and Zin show that γ governs the coefficient of relative risk aversion and ψ the elasticity of intertemporal substitution. These preferences thus have the useful property that it is possible to separate the agent's relative risk aversion from the elasticity of intertemporal substitution, unlike the standard power utility case where $\gamma = \frac{1}{\psi}$. If $\gamma \neq \frac{1}{\psi}$, the utility function is no longer time-additive and agents care about the temporal distribution of risk - a feature that is central to our analysis. We discuss this property and its implications in more detail below.

The Stochastic Discount Factor and Risk. The stochastic discount factor, M_{t+1} , is the ratio of the representative agent's marginal utility between today and tomorrow: $M_{t+1} = \frac{U'(C_{t+1})}{U'(C_t)}$. Using a recursive argument, Epstein and Zin (1989) show that:

$$\ln M_{t+1} \equiv m_{t+1} = \theta \ln \beta - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) r_{a,t+1}, \quad (2)$$

where $\Delta c_{t+1} \equiv \ln \frac{C_{t+1}}{C_t}$ and $r_{a,t+1} \equiv \ln \frac{A_{t+1} + C_{t+1}}{A_t}$ is the return on the total wealth portfolio with A_t denoting total wealth at time t .⁵ If $\gamma = \frac{1}{\psi}$, $\theta = \frac{1-\gamma}{1-1/\psi} = 1$, and the stochastic discount factor collapses to the familiar power utility case, where shocks to realized consumption growth are the only source of risk in the economy. However, if $\gamma \neq \frac{1}{\psi}$, the return on the wealth portfolio appears as a risk factor. Persistent time-variation in expected consumption growth (the expected "dividends" on the total wealth portfolio) induces higher volatility of asset returns (Barsky and DeLong, 1993). Thus, the return on any asset is a function of the dynamic behavior of realized *and* expected consumption growth (Bansal and Yaron, 2004).

⁵Note that our representative household's total wealth portfolio is composed of the present value of future labor income in addition to the value of the firm.

Depending on the sign of θ and the covariance between realized consumption growth and the return on the total wealth portfolio, the volatility of the stochastic discount factor (i.e., the price of risk in the economy) can be higher or lower relative to the benchmark power utility case (see the appendix for further discussion). We show later how this covariance, and thus the amount of long-run risk due to endogenous consumption smoothing, changes with the persistence of the technology shock.

Technology. There is a representative firm with a Cobb-Douglas production technology:

$$Y_t = (Z_t H_t)^{1-\alpha} K_t^\alpha, \quad (3)$$

where Y_t denotes output, K_t the firm's capital stock, H_t the number of hours worked, and Z_t denotes the (stochastic) level of aggregate technology. This constant returns to scale and decreasing marginal returns production technology is standard in the macroeconomic literature. Since we assume leisure not to enter the utility function, households incur no disutility of working and supply a constant amount of hours worked (as in, e.g., Jermann, 1998). We normalize $H_t = 1$. The productivity of capital and labor depends on the level of technology, Z_t , which is the exogenous driving process of the economy. We model log technology, $z \equiv \ln(Z)$, as:

$$z_t = \mu t + \tilde{z}_t, \quad (4)$$

$$\tilde{z}_t = \varphi \tilde{z}_{t-1} + \sigma_z \varepsilon_t, \quad (5)$$

$$\varepsilon_t \sim N(0, 1), \quad |\varphi| \leq 1. \quad (6)$$

Thus, (5) implies that technology shocks are permanent if $\varphi = 1$ and transitory if $\varphi < 1$. Both specifications are commonly used in the literature.⁶ We discuss these two cases separately because they have very different implications for asset prices and macroeconomic time series.

Capital Accumulation and Adjustment Costs. The agent can shift consumption from today to tomorrow by investing in capital. The firm accumulates capital according to

⁶See, for example, Campbell (1994), who considers permanent and transitory, Cooley and Prescott (1995), transitory, Jermann (1998), permanent and transitory, Prescott (1986), permanent, Rouwenhorst (1995), permanent and transitory.

the following law of motion:

$$K_{t+1} = (1 - \delta) K_t + \phi\left(\frac{I_t}{K_t}\right) K_t, \quad (7)$$

where I_t is aggregate investment and $\phi(\cdot)$ is a positive, concave function, capturing the notion that adjusting the capital stock rapidly by a large amount is more costly than adjusting it step by step. We follow Jermann (1998) and Boldrin, Christiano, Fisher (1999) and specify:

$$\phi(I_t/K_t) = \frac{\alpha_1}{1 - 1/\xi} \left(\frac{I_t}{K_t}\right)^{(1-1/\xi)} + \alpha_2, \quad (8)$$

where α_1, α_2 are constants and $\alpha_1 > 0$. The adjustment cost specification implies that equilibrium aggregate investment is positive, and α_1 and α_2 are set so that the firm does not incur adjustment costs when investing at the steady state rate.⁷ The parameter ξ is the elasticity of the investment-capital ratio with respect to Tobin's q . If $\xi = \infty$ the capital accumulation equation reduces to the standard growth model accumulation equation without capital adjustment costs.

Each period the firm's output, Y_t , can be used for either consumption or investment. Investment increases the firm's capital stock, which in turn increases future output. High investment, however, means the agent must forego some consumption today ($C_t = Y_t - I_t$).

The Return on Investment and the Firm's Problem. Let $\Pi(K_t, Z_t; W_t)$ be the operating profit function of the firm, where W_t are equilibrium wages.⁸ Firm dividends, D_t , equal operating profits minus investment:

$$D_t = \Pi(K_t, Z_t; W_t) - I_t. \quad (9)$$

The firm maximizes firm value. Let $M_{t,t+1}$ denote the stochastic discount factor. The firm's problem is then:

$$\max_{\{I_t, K_{t+1}, H_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} M_{0,t} D_t, \quad (10)$$

⁷In particular, we set $\alpha_1 = (\exp(\mu) - 1 + \delta)^{1/\xi}$ and $\alpha_2 = \frac{1}{\xi-1} (1 - \delta - \exp(\mu))$. It is straightforward to verify that $\phi(\frac{I_t}{K_t}) > 0$ and $\phi''(\frac{I_t}{K_t}) < 0$ for $\xi > 0$ and $\frac{I_t}{K_t} > 0$. Furthermore, $\phi(\frac{I}{K}) = \frac{I}{K}$ and $\phi'(\frac{I}{K}) = 1$, where $\frac{I}{K} = (\exp(\mu) - 1 + \delta)$ is the steady state investment-capital ratio. Investment is always positive since the marginal cost of investing goes to infinity as investment goes to zero.

⁸Wages are in the first part of the paper assumed to be the marginal productivity of labor: $W_t = (1 - \alpha) Y_t$. Since $C_t = D_t + W_t$, it follows that $D_t = \alpha Y_t - I_t$.

where E_t denotes the expectation operator conditioning on information available up to time t . In the appendix, we demonstrate that the return on investment can be written as:

$$R_{t+1}^I = \phi' \left(\frac{I_t}{K_t} \right) \left(\Pi_K (K_{t+1}, Z_{t+1}; W_{t+1}) + \frac{1 - \delta + \phi \left(\frac{I_{t+1}}{K_{t+1}} \right)}{\phi' \left(\frac{I_{t+1}}{K_{t+1}} \right)} - \frac{I_{t+1}}{K_{t+1}} \right). \quad (11)$$

This return to the firm's investment is equivalent to the firm's equity return in equilibrium, $R_{t+1}^E \equiv \frac{D_{t+1} + P_{t+1}}{P_t}$, where P_t denotes the net present value of a claim on all future dividends (see, e.g., Restoy and Rockinger, 1994, and Zhang, 2005).

4 Results

The model generates macroeconomic aggregates such as output, investment, and consumption, in addition to the standard financial moments. In the first part of the analysis, we present two long-run risk calibrations of the model corresponding to transitory and permanent technology shocks, respectively. Then we explain the intuition for the endogenous long-run risk we document by analyzing the endogenous, dynamic behavior of consumption growth. Our discussion is centered around different values of the elasticity of intertemporal substitution and the two specifications of technology (permanent vs. transitory). We subsequently discuss the asset pricing implications of the model in more detail. We solve the model numerically by means of the value function iteration algorithm. Please refer to the appendix for a detailed discussion of our solution technique.

4.1 Calibration

We report calibrated values of model parameters that are constant across models in Table 1. The capital share (α), the depreciation rate (δ), and the mean technology growth rate (μ) are set to standard values for quarterly parametrizations (see, e.g., Boldrin, Christiano, and Fisher, 2001). We consider two values for the persistence of the technology shocks, $\varphi \in \{0.95, 1\}$, which are both commonly used in the literature.⁹

We set the coefficient of relative risk aversion (γ) to 5 across all models in the main part of the paper. This value is in the middle of the range of reasonable values for the coefficient of relative risk aversion, as suggested by Mehra and Prescott (1985). The focus of our paper

⁹See Prescott (1986) for a discussion of the empirical persistence of Solow residuals.

Table 1
Calibration

Table 1: Calibrated values of parameters that are constant across models.

Quarterly Model Calibration		
Parameter	Description	Value
α	Elasticity of capital	0.34
δ	Depreciation rate of capital	0.021
γ	Coefficient of relative risk aversion	5
μ	Mean technology growth rate	0.4%
φ	Persistence of the technology shock	{0.95, 1}

is on the role of endogenous long-run risk arising from optimal consumption smoothing, which in turn is largely determined by the elasticity of intertemporal substitution. Tallarini (2000) analyses a similar model (without capital adjustment costs), but instead assumes a fixed elasticity of intertemporal substitution and varies the level of relative risk aversion. He finds, and we confirm his finding (see appendix), that macroeconomic time series are almost unaffected by the level risk aversion, holding the elasticity of intertemporal substitution constant.

We vary the elasticity of intertemporal substitution (ψ) across models and use the capital adjustment costs (ξ) to fit (if possible) the relative volatility of consumption to output. The discount factor (β) is set to match the level of the risk free rate. We vary the volatility of technology shocks (σ_z) in order to fit the empirical consumption growth volatility with all models. We discuss the choice of specific parameter values for these variables as we go along.

4.2 Two Models with Long-Run Risk

We preview our results by showing two calibrations of the model which both, because of endogenous long-run risk, can match the low volatility of consumption growth and the high price of risk with a low coefficient of relative risk aversion. The key distinctions between the models are the difference in the elasticity of intertemporal substitution and the persistence of the technology shocks. The transitory technology shock model has a low (0.05) elasticity of intertemporal substitution, whereas the permanent shock model has a high (1.5) elasticity of intertemporal substitution.

Panel A of Table 2 shows that both models match the volatility of consumption, the

Table 2
Asset Pricing Moments: Adjusted Model versus a Standard Model

Table 2: This table reports annual asset pricing moments for two calibrations of the stochastic growth model where the representative agent has Epstein-Zin preferences and there are adjustment costs to capital. The models have permanent and transitory technology shocks, respectively. The level of risk aversion (γ) is 5 in both models, and the volatility of shocks to technology, σ_z , is the same for both models. The volatility of shocks to technology is calibrated such that the models fit the volatility of consumption growth. Both models are calibrated to match the relative volatility of consumption to output, the volatility of output, the level of the risk free rate, and the Sharpe ratio of equity returns. The equity returns in both models are for an unlevered claim on the endogenous, aggregate dividends. The equity premium due to "short-run" risk is defined as $\gamma cov(\Delta c_t, R_t^E - R_{f,t})$. The empirical moments are taken from the annual U.S. sample from 1929-1998, corresponding to the sample in Bansal and Yaron (2004).

Statistic	U.S. Data 1929 – 1998	Transitory shocks: $\varphi = 0.95$	Permanent shocks: $\varphi = 1$
		Long-Run Risk I $\psi = 0.05, \gamma = 5$ $\beta = 1.064, \xi = 0.52$	Long-Run Risk II $\psi = 1.5, \gamma = 5$ $\beta = 0.998, \xi = 18$
Panel A - Calibrated Moments			
<i>Volatility of Consumption Growth</i> $\sigma [\Delta c]$ (%)	2.72	2.72	2.72
<i>Relative Volatility of Consumption and Output (GDP)</i> $\sigma [\Delta c] / \sigma [\Delta y]$	0.52	0.52	0.52
<i>Level of Risk Free Rate</i> $E [R_f]$ (%)	0.86	0.85	0.82
<i>Sharpe ratio of Equity Returns</i> $E [R^E - R_f] / \sigma [R^E - R_f]$	0.33	0.34	0.36
Panel B - Other Moments			
<i>Volatility of the Risk Free Rate</i> $\sigma [R_f]$ (%)	0.97	4.60	0.45
<i>Equity Returns</i> $E [R^E - R_f]$ (%)	6.33	8.06	0.24
$\sigma [R^E - R_f]$ (%)	19.42	24.06	0.66
<i>Decomposing the Equity Premium (%)</i>			
<i>Short-Run Risk</i>		3.27 (41%)	0.09 (38%)
<i>Long-Run Risk</i>		4.79 (59%)	0.15 (62%)

relative volatility of consumption to output, the level of the risk free rate, and the Sharpe ratio of the aggregate claim to dividends. The latter fact is remarkable with a coefficient of relative risk aversion of only 5! With a consumption volatility of 2.72%, a power utility model would give a Sharpe ratio of only 0.14, whereas both calibrations of the Epstein-Zin model match the sample annual Sharpe ratio of 0.33. Thus, both models generate a substantial amount of endogenous long-run risk in the consumption process, although the way in which they do so is quite different. In particular, in the model with a low EIS, the representative agent has a preference for late resolution of uncertainty ($\gamma < \frac{1}{\psi}$), whereas in the model with a high EIS, the representative agent has a preference for early resolution of uncertainty ($\gamma > \frac{1}{\psi}$). It is important to note that the transitory shock model matches the risk free rate by specifying a discount rate (β) greater than one. Prices in this economy are still well-defined, however, since the economy is growing (see Kocherlakota, 1990). Some may principally object to a value of β greater than one. If we were to restrict $\beta < 1$, the risk free rate in the transitory shock model would increase to over 25% on an annual basis (the risk free rate puzzle). The permanent shock model, however, has $\beta = 0.998$, so it is not subject to this problem.

Panel B shows financial moments the models were not calibrated to fit. The transitory shock model displays too high volatility of the risk free rate, since agents in this economy are very unwilling to substitute consumption across time. The equity claim is defined as the (unlevered) claim to aggregate dividends. The equity return volatility is higher than in the data and the equity premium is therefore also higher. This is quite the opposite of what we are used to from production economies, which are generally deemed to not be able to produce any kind of sizeable equity premium. The reason is that capital adjustment costs are set quite high in this economy to fit the relative volatility of consumption growth to output growth. Panel B reports that about 60% of the risk premium is due to long-run risk, where short-run risk is defined as $\gamma Cov(R_t^E - R_{f,t}, \Delta c_t)$.

In the permanent shock model, the risk free rate displays low volatility, as in the data, despite the high price of risk. This feature is a marked improvement over habit formation models in production economies, which can match the price of risk, but generate much too volatile risk free rates (see, e.g., Jermann, 1998, and Boldrin, Christiano, and Fisher, 2001). Since the reciprocal of the risk free rate is the conditional expectation of the stochastic discount factor, mismatching the risk free rate volatility implies mismatching the dynamic behavior of the stochastic discount factor. In this model, however, the equity return volatility and therefore the risk premium are too low. This is because capital adjustment costs must

be very low in order to match the relative volatility of consumption growth when technology shocks are permanent. Again, about 60% of the risk premium is due to long-run risk.

In sum, *both* models generate substantial amount of long-run risk in the endogenous consumption process. Over the next sections, we analyze the mechanisms within the model that give rise to these results. We also suggest a simple extension of the permanent shock model which increases the equity premium in this model by an order of magnitude.

4.3 The Endogenous Consumption Choice and The Price of Risk

Before we report moments from different calibrations of the model, it is useful to provide some general intuition for the endogenous consumption choice and how it is related to the persistence of the technology shocks and the price of risk in the economy. From the stochastic discount factor (see eq. (2)), we can see that there are two sources of risk in this economy. The first is the shock to realized consumption growth, which is the usual risk factor in the Consumption CAPM. The second risk factor is the shock to the return to total wealth. Total wealth is the sum of human and financial capital, and the dividend to total wealth is consumption. Assume for the moment that future expected consumption growth and returns are constant. Total wealth, A_t , is then given by:

$$A_t = \frac{C_t}{r_a - g_c}, \quad (12)$$

where r_a is the expected return to wealth and g_c is expected consumption growth: Total wealth is a function of both current and future expected consumption. Therefore, shocks to both realized and expected consumption growth translate into shocks to the realized return to wealth. This example illustrates how we can think of shocks to expected consumption growth as the second risk factor instead of the return to wealth.¹⁰ Understanding the dynamic behavior of consumption growth is thus necessary in order to understand the asset pricing properties of the production economy model with Epstein-Zin preferences. In the following, we consider the consumption response to both transitory and permanent technology shocks.¹¹

¹⁰Following Bansal and Yaron (2004), we explicitly show this in the appendix through a log-linear approximation of the return to wealth.

¹¹We make a strong distinction between transitory and permanent shocks in this section in order to provide clear intuition. As $\varphi \rightarrow 1$, the transitory shock specification (5) approaches the permanent shock specification (4). The dynamics of the model are in that case very similar for both specifications, so there is actually no discontinuity at $\varphi = 1$ in terms of the model's asset pricing implications. However, the transitory

Figure 1 - Transitory and Permanent Shocks

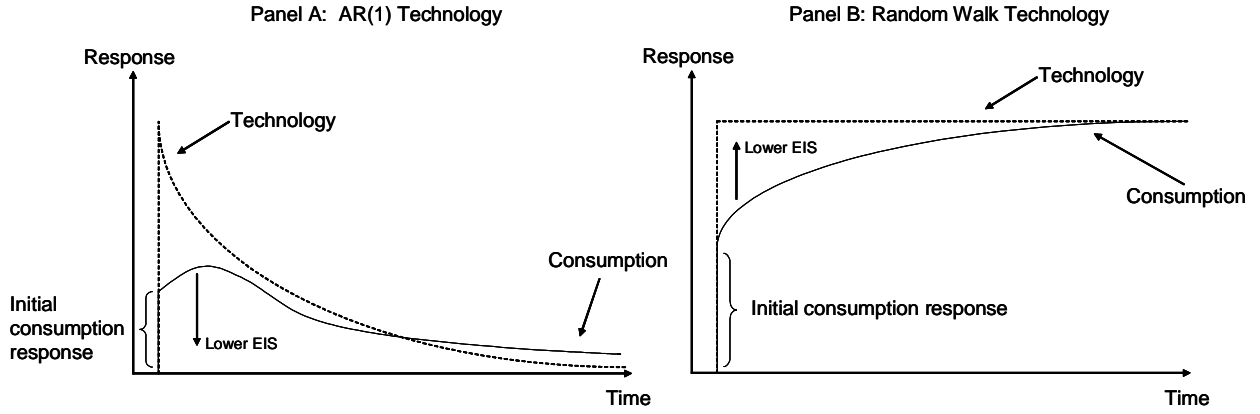


Figure 1: **Impulse-Responses for Technology and Consumption.** Panel A shows the impulse-response of technology and consumption to a transitory technology shock. Panel B shows the impulse-response of technology and consumption to a permanent technology shock. The arrows show the direction in which the optimal consumption response changes if the desire for a smoother consumption path increases (i.e., the elasticity of intertemporal substitution decreases).

Transitory Technology Shocks. Panel A of Figure 1 shows the impulse-response functions of technology and consumption to a transitory technology shock. Agents in this economy want to take advantage of the temporary increase in the productivity of capital due to the temporarily high level of technology. To do so, they invest immediately in capital at the expense of current consumption. As a result, the consumption response is hump-shaped. This figure illustrates how time-varying expected consumption growth arises endogenously in the production economy model: A *positive* shock to realized consumption growth (the initial consumption response) is associated with positive short-run expected consumption growth, but *negative* long-run expected consumption growth as consumption reverts back to the steady state. Thus, the shock to long-run expected consumption growth is negatively correlated with the shock to realized consumption growth.

Permanent Technology Shocks. With permanent technology shocks, long-run consumption risk has the opposite effect. Panel B of Figure 1 shows the impulse-response functions of technology and consumption to a permanent technology shock. Technology

shocks need to be extremely persistent for the transitory and permanent cases to be similar. At $\varphi = 0.95$, which is the case we consider in our calibration, the dynamic behavior of the model with permanent shocks is very different from the model with transitory shocks. The reader could therefore think of "transitory vs. permanent" shocks as "not extremely persistent vs. extremely persistent" shocks.

adjusts immediately to the new steady state, and the permanently higher productivity of capital implies that the optimal long-run levels of both capital and consumption are also higher. Agents invest immediately in order to build up capital at the expense of current consumption, and consumption gradually increases towards the new steady state after the initial shock. Thus, a positive shock to realized consumption growth (the initial consumption response) is associated with *positive* long-run expected consumption growth. In this case, the two shocks are therefore positively correlated.

The Elasticity of Intertemporal Substitution. The elasticity of intertemporal substitution (*EIS*) is an important determinant of the dynamic behavior of consumption growth. A low *EIS* translates into a strong desire for intertemporally smooth consumption paths. In other words, agents strive to minimize the difference between their level of consumption today (after the shock) and future expected consumption levels. The arrows in Figure 1 indicate the directions in which the initial optimal consumption responses change if the desire for a smoother consumption path increases. As the elasticity of intertemporal substitution decreases, agents desire a "flatter" response curve. From the figure, we can conjecture that a lower *EIS* decreases the volatility of expected future consumption growth. A high *EIS*, on the other hand, implies a higher willingness to substitute consumption today for higher future consumption levels. Therefore, the higher the *EIS*, the higher the volatility of expected consumption growth and the higher the levels of long-run risk in the economy.

Capital Adjustment Costs. Capital adjustment costs (*CAC*) make it more costly for firms to adjust investment. Therefore, higher *CAC* induce lower investment volatility. We can therefore use *CAC* in order to, as far as possible, match the empirical relative volatilities of consumption, investment, and output with each model.

Implications for the Price of Risk. The log return to wealth can be written as:

$$r_{a,t+1} = \Delta c_{t+1} + \tilde{r}_{a,t+1}, \quad (13)$$

where $\tilde{r}_{a,t+1} = \log \left(1 + \frac{A_{t+1}}{C_{t+1}} \right) - \log \frac{A_t}{C_t}$. Shocks to this "adjusted" wealth return reflect only updates in expectations about future consumption growth and discount rates (see, e.g.,

Campbell and Shiller, 1988). Now, write the stochastic discount factor as:

$$\begin{aligned}
m_{t+1} &= \theta \ln \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{a,t+1} \\
&= \theta \ln \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) \Delta c_{t+1} + (\theta - 1) \tilde{r}_{a,t+1} \\
&= \theta \ln \beta - \gamma \Delta c_{t+1} + (\theta - 1) \tilde{r}_{a,t+1}.
\end{aligned} \tag{14}$$

Since there is only one shock in this economy (the technology shock), we can write:

$$\Delta c_{t+1} = E_t [\Delta c_{t+1}] + \sigma_{\Delta c,t} \varepsilon_{t+1}, \tag{15}$$

and:

$$\tilde{r}_{a,t+1} = E_t [\tilde{r}_{a,t+1}] + \sigma_{\tilde{r},t} \varepsilon_{t+1}, \tag{16}$$

where ε_{t+1} is the technology shock. Note that the parameters $\sigma_{\Delta c,t}$ and $\sigma_{\tilde{r},t}$ multiplying the technology shock can be positive or negative, depending on the correlation. Innovations to the log stochastic discount factor are then:

$$m_{t+1} - E_t [m_{t+1}] = (-\gamma \sigma_{\Delta c,t} + (\theta - 1) \sigma_{\tilde{r},t}) \varepsilon_{t+1}. \tag{17}$$

Define the price of risk as the conditional volatility of the log stochastic discount factor:

$$\begin{aligned}
\Lambda_t &= \gamma \sigma_{\Delta c,t} + (1 - \theta) \sigma_{\tilde{r},t} \\
&= \gamma \sigma_{\Delta c,t} + (\gamma - 1/\psi) (1 - 1/\psi)^{-1} \sigma_{\tilde{r},t}.
\end{aligned} \tag{18}$$

In our calibrations, shocks to consumption growth are positively correlated with shocks to technology. Therefore, we have $\sigma_{\Delta c,t} > 0$. Below we consider the nature of the price of risk in this model for different attitudes to the resolution of uncertainty and persistence of the technology shocks.

1. If $\gamma = \frac{1}{\psi}$, the preferences are standard additive expected utility: $\theta = 1$ and $\Lambda_t = \gamma \sigma_{\Delta c,t}$.
2. If consumption growth is i.i.d. then, regardless of the preference parameters, $\sigma_{\tilde{r},t} = 0$ and $\Lambda_t = \gamma \sigma_{\Delta c,t}$, as in the power utility model (in this case, both growth rates and discount rates are constant, so the wealth-consumption ratio is constant).
3. Now we turn to the relevant case where consumption growth is not i.i.d. and agents

are not indifferent to the timing of the resolution of uncertainty.

Whether the wealth-consumption ratio responds positively or negatively to technology shocks (i.e., whether $\sigma_{\tilde{r},t} \leq 0$), depends on both the persistence of the shocks and on whether the substitution or the income effect dominates (i.e., whether $\psi \geq 1$). Furthermore, $(1 - 1/\psi)$ also changes sign depending on whether $\psi \geq 1$. Thus, the product $(1 - 1/\psi)^{-1} \sigma_{\tilde{r},t}$ only depends on the persistence of the technology process. We use this observation to consider four general cases with different implications for the price of risk:

(a) Agents prefer early resolution of uncertainty ($\gamma > 1/\psi$):

- i. **Permanent technology shocks:** In this case, a positive technology shock leads to a positive shock to expected consumption growth (see previous discussion and figure 1) and $(1 - 1/\psi)^{-1} \sigma_{\tilde{r},t} > 0$. Therefore, shocks to realized consumption growth and shocks to the adjusted wealth return reinforce each other and the price of risk is *higher* relative to the power utility case. As an example, consider the case where the substitution effect dominates, i.e., $(1 - 1/\psi)^{-1} > 0$. Then $\sigma_{\tilde{r},t} > 0$ since the shock to the wealth-consumption ratio is dominated by the positive shock to expected consumption growth.
- ii. **Transitory technology shocks:** In this case, long run expected consumption growth after a positive technology shock is *negative* as consumption must revert back to the trend and $(1 - 1/\psi)^{-1} \sigma_{\tilde{r},t} < 0$. Therefore, shocks to the adjusted wealth return and shocks to realized consumption growth hedge each other and the price of risk is *lower* relative to the power utility case. As an example, consider the case where the substitution effect dominates, (i.e., $(1 - 1/\psi)^{-1} > 0$). Then $\sigma_{\tilde{r},t} < 0$ since the shock to the wealth-consumption ratio is dominated by the negative shock to expected consumption growth.

For agents who prefer early resolution of uncertainty, transitory technology shocks are less risky than permanent technology shocks.

(b) Agents prefer late resolution of uncertainty ($\gamma < 1/\psi$):

- i. **Permanent technology shocks:** As in the permanent shock case above, $(1 - 1/\psi)^{-1} \sigma_{\tilde{r},t} > 0$. However, since $\gamma < 1/\psi$, shocks to the adjusted wealth return and shocks to realized consumption growth now hedge each other and the price of risk is lower relative the power utility case.

- ii. **Transitory technology shocks:** As in the transitory shock case above, $(1 - 1/\psi)^{-1} \sigma_{\tilde{r},t} < 0$. However, since $\gamma < 1/\psi$, shocks to the adjusted wealth return and shock to realized consumption growth now reinforce each other and the price of risk is *higher* relative to the power utility case.

For agents who prefer late resolution of uncertainty, transitory technology shocks are more risky than permanent technology shocks.

In order to generate a high price of risk, which is the empirically relevant case, we either need a preference for early resolution of uncertainty and permanent technology shocks, or a preference for late resolution of uncertainty and transitory technology shocks.

What can agents do to endogenously decrease these risks? Very little. While the agents will attempt to endogenously make the consumption risk as small as possible, they cannot easily get rid of it. Consider the permanent shock case displayed in figure 2. Agents can perform more consumption smoothing in order to decrease the volatility of realized consumption growth (dashed line). However, decreasing the shock to realized consumption growth increases the shock to expected consumption growth which with Epstein-Zin preferences is also a priced risk factor. Thus, unlike in the power utility model, the agents are caught in a Catch-22: Decreasing one risk increases another.

In the following section, we show how the above developed intuition manifests itself quantitatively.

4.3.1 Results from Calibrated Models

Table 3 confirms the intuition from the impulse-responses in figures 1 and 2 by reporting relevant macroeconomic moments and the equilibrium price of risk for different model calibrations. The models have either transitory or permanent technology shocks and different levels of the elasticity of intertemporal substitution ($\psi \in \{0.05, 1/\gamma = 0.2, 1.5\}$). Given these, we match the relative volatility of consumption and output growth (if possible) by changing the capital adjustment cost (ξ). We match the volatility of consumption growth with all models by setting the volatility of the technology shocks, σ_z , appropriately. We re-calibrate the discount factor (β) for each model to match the level of the risk free rate, as far as possible.¹² The coefficient of relative risk aversion ($\gamma = 5$) is constant across models. We

¹²We do not restrict β to be less than one, but if β becomes too large, prices are not well-defined (they are infinite).

Table 3

Macroeconomic Moments and Consumption Dynamics

Table 3: This table reports relevant macroeconomic moments and consumption dynamics for models with either transitory ($\varphi = 0.95$) or permanent ($\varphi = 1.00$) technology shocks and different levels of the elasticity of intertemporal substitution (ψ). The coefficient of relative risk aversion (γ) is 5 across all models. We calibrate the discount factor (β) for each model to match the level of the risk free rate. The volatility of the technology shock (σ_z) is set so that the volatility of consumption growth is the same across models. Capital adjustment costs (ξ) are set so that (if possible) the relative volatility of consumption to output growth is matched. We use annual U.S. data from 1929 to 1998 from the Bureau of Economic Analysis for the empirical moment values. The sample is the same as in Bansal and Yaron (2004).

Statistic	LRR I		LRR II	
	Model 2	Model 3	Model 4	Model 5
	Transitory Shocks $\varphi = 0.95, \gamma = 5$		Permanent Shocks $\varphi = 1.00, \gamma = 5$	
	$\psi = 0.05$	$\psi = 1.5$	$\psi = 0.05$	$\psi = \frac{1}{\gamma}$
	$\beta = 1.064$	$\beta = 0.998$	$\beta = 1.067$	$\beta = 1.012$
	$\xi = 0.70$	$\xi = 6.75$	$\xi = \infty$	$\xi = \infty$
	$\sigma_z = 2.59\%$	$\sigma_z = 2.59\%$	$\sigma_z = 1.03\%$	$\sigma_z = 1.20\%$
				$\sigma_z = 2.63\%$

Panel A: Macroeconomic Moments (Quarterly)

	U.S. Data 1929-1998		Bansal, Yaron Calibration	
$\sigma[\Delta y]$ (%)	2.61	2.61	1.03	1.20
$\sigma[\Delta c]/\sigma[\Delta y]$	0.52	0.52	1.32	1.13
$\sigma[\Delta \tilde{v}]/\sigma[\Delta y]$	2.36	1.86	0.34	0.75
$AdjCosts/Output$ (%)	n/a	0.31	0.00	0.00
				2.64
				0.52
				1.83
				0.04

Panel B: Consumption Dynamics:

	Bansal, Yaron Calibration		Bansal, Yaron Calibration	
$\sigma[\Delta c_t]$ (%)	1.360	1.360	1.360	1.360
$\sigma[E_t[\Delta c_{t+1}]]$ (%)	0.172	0.144	0.013	0.040
$Autocorr(E_t[\Delta c_{t+1}])$	0.938	0.962	0.995	0.988
				1.362
				0.336
				0.970

Panel C: The Price of Risk and the Sharpe ratio of the Equity Return (Annual)

	Bansal, Yaron Calibration		Bansal, Yaron Calibration	
$\sigma[M]/E[M]$	n/a	0.14	0.08	0.14
$SR[R^E]$	0.33	0.14	0.08	0.14
				0.36
				0.36

Figure 2 - The Catch-22 of Consumption Smoothing

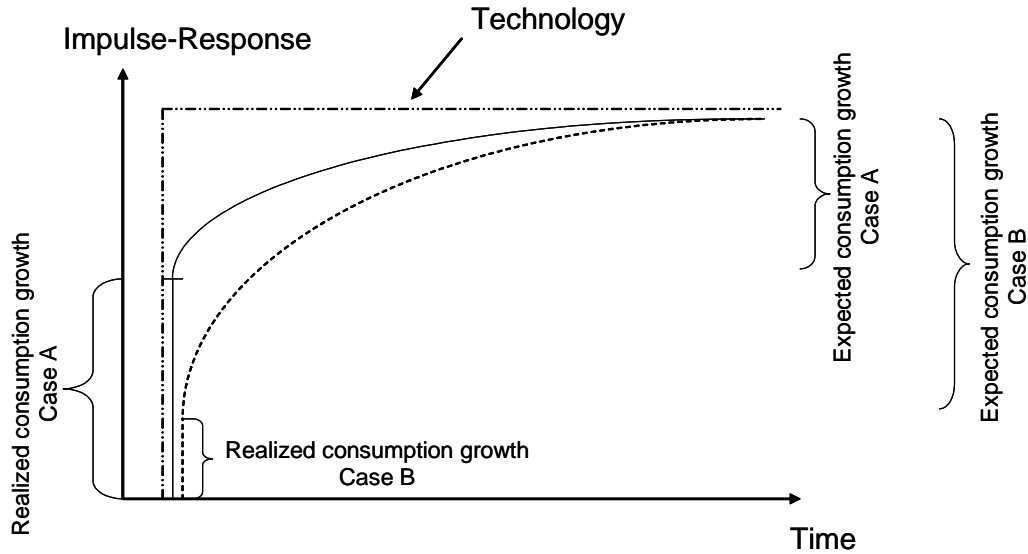


Figure 2: **Impulse-Responses for Technology and Consumption.** The dash-dotted line shows the impulse-response of the level of technology to a permanent technology shock. The solid line shows the impulse response of consumption when agents choose the initial consumption response to be large (Case A). The dashed line shows the consumption response when agents choose the initial consumption response to be small (Case B). The graph shows that by making the shock to realized consumption small, the shock to expected consumption growth becomes large.

show in the appendix, confirming Tallarini (2000), that the level of γ has only second-order effects on the time series behavior of the macroeconomic variables.

The Macroeconomic Moments. All the models with transitory technology shocks match both the volatility of consumption and output growth. For low levels of the *EIS*, the agents strongly desire a smooth consumption path and therefore would like to decrease the consumption response to a transitory technology shock by investing more. To prevent this from happening, we increase the capital adjustment costs. The table reports that capital adjustment costs for the model with the lowest *EIS* are on average 0.88% of output. These high adjustment costs are the reason that asset prices are very volatile, as we show in the next section. The permanent shock model, on the other hand, cannot, even with no capital adjustment costs, match both the volatility of output and consumption growth unless the *EIS* is high (1.5). With permanent shocks, a strong desire to smooth consumption means agents want the initial consumption response to be *high* and close to its new, higher steady state level. Capital adjustment costs decrease investments, which otherwise would make the

consumption response less strong. But, even in the case of no capital adjustment costs, the investment response is not strong enough unless the *EIS* is sufficiently high.

The Volatility of Expected Consumption Growth (Long-run Risk). In Panel B of Table 3, we report both the volatility of consumption growth, the volatility of conditional expected consumption growth, $\sigma [E_t [\Delta c_{t+1}]]$, and the latter's first order autocorrelation (ρ). These statistics illustrate the magnitude and nature of long-run risk in the models.¹³ For comparison, Panel B also gives the corresponding values that Bansal and Yaron (2004) use in their calibration. The relative magnitudes of the volatility of realized and expected consumption growth show that the time-varying growth component is small. The implied average R^2 across models is around 1 – 2%, with a maximum R^2 of 6% for the model with permanent shocks and *EIS* of 1.5 (LLR II). Note that the R^2 of an AR(1) on consumption growth would be much lower as realized consumption growth is a noisy measure of expected consumption growth. The persistence of the expected consumption growth rate (ρ) is very high, which is important if risk associated with a small time-varying expected consumption growth rate component is to have quantitatively interesting asset pricing implications.

The Price of Risk. Even though the coefficient of relative risk aversion and the volatility of consumption growth are the same across all models, the price of risk varies from close to zero to 0.36. The power utility calibrations both give a price of risk of 0.14, and deviations from this value are due to the effect of long-run risk in the model. In the case of transitory technology shocks, the price of risk is decreasing in the *EIS*. Holding γ constant and increasing the *EIS* increases the preference for early resolution of uncertainty, in which case agents dislike shocks to expected consumption growth: In Model 3, the price of risk is low because the two risk factors, shocks to realized and expected future consumption growth, are negatively correlated and therefore hedge each other. In the first Long-Run Risk model (LLR I), the agents instead prefer late resolution of uncertainty and therefore *like* shocks to expected consumption growth. For these agents, a world where shocks to realized consumption (which they dislike) and expected consumption (which they like) are negatively correlated, is a more risky world. That is why the price of risk in this case is high. The same logic applies for the case of permanent shocks, where the two shocks to consumption

¹³In the appendix, we show that these moments indeed capture most of the dynamics of consumption growth generated by the models and as such are meaningful moments to consider. There is some heteroskedasticity in both shocks to expected and realized consumption growth, but these effects are second order.

are positively correlated. In this case, it is the high *EIS* model (LLR II) that has a high price of risk.

Figure 3 - Impulse Response 1

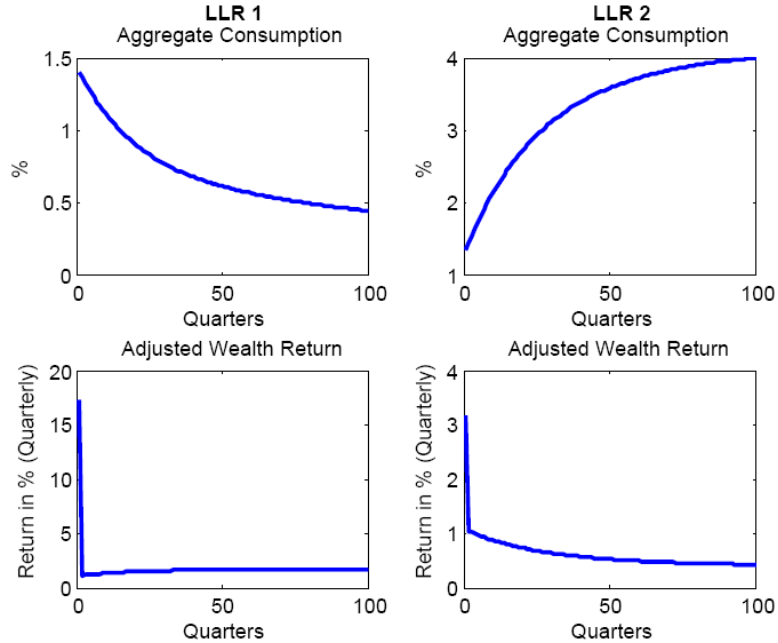


Figure 3: **Impulse-Responses for Consumption and the Adjusted Wealth Return.** The plots show the impulse-responses of consumption and the adjusted wealth return for the LLR I (transitory technology shocks) and the LLR II (permanent technology shocks).

This intuition is confirmed by figure 3, which shows the impulse-response of both consumption and the adjusted return to wealth $\left(\tilde{r}_{a,t} = \log\left(1 + \frac{A_{t+1}}{C_{t+1}}\right) - \log\frac{A_t}{C_t}\right)$. Remember from the above discussion that the price of risk is:

$$\Lambda_t = \gamma\sigma_{\Delta c,t} + (\gamma - 1/\psi)(1 - 1/\psi)^{-1}\sigma_{\tilde{r},t}. \quad (19)$$

For both the long-run risk model with transitory technology shocks and low *EIS* ("LLR I") and the long-run risk model with permanent technology shocks and a high *EIS* ("LLR II"), $(\gamma - 1/\psi)(1 - 1/\psi)^{-1} > 0$. The figure shows that the response of the adjusted wealth return is the same. In the first case, the long-run expected consumption growth is negative, but since $\psi < 1$, the income effect dominates and the wealth-consumption ratio increases. In the second case, the long-run expected consumption growth is positive, and since $\psi > 1$,

the substitution effect dominates. Thus, the wealth to consumption ratio increases here too and $(\gamma - 1/\psi)(1 - 1/\psi)^{-1} \sigma_{\bar{r}} > 0$.

4.4 Asset Pricing Implications

Table 4 presents key financial moments for the same models as in Table 3.

The Risk Free Rate. The level of the risk free rate is decreasing in the *EIS*, all else equal. A higher *EIS* increases the intertemporal substitution effect, which increases the demand for bonds in a growing economy.¹⁴ For the transitory shock models this effect can be countered by a very high discount factor (β), which we do not restrict to be less than one in this paper. If we impose this restriction ($\beta < 1$), the risk free rate puzzle obtains. Note that we cannot simply increase β without bound, as the equilibrium prices then do not converge. While one can debate whether the magnitude of β is acceptable or not, the models with low *EIS* have in any case a too high volatility of the risk free rate. Since the risk free rate is the reciprocal of the conditional expected value of the stochastic discount factor, a misspecified risk free rate implies a misspecified stochastic discount factor. Habit formation models typically encounter this problem (see, e.g., Jermann (1998) or Boldrin, Christiano and Fisher (2001), as time-variation in the state variable "surplus consumption" induces much too volatile risk free rates when the models are calibrated to match empirical proxies for the price of risk (e.g., the equity Sharpe ratio). In contrast, the permanent shock model with high *EIS* (LLR II), can match both the level and the volatility of the risk free rate, as well as a high price of risk.

The Consumption Claim. Aggregate wealth is the value of the claim to the aggregate consumption stream. The return volatility of the consumption claim is strongly increasing in capital adjustment costs. This is a well known feature of this friction in the case of the return to equity as it allows marginal q to deviate from 1. For model "LLR I" (the transitory shock case), the volatility of the consumption claim is very high at 29.15% per year. Model "LLR II", which has much lower capital adjustment costs, displays a return volatility of the consumption claim of 4.41%.

¹⁴See eq. (48) in the appendix, for an approximate expression for the risk free rate.

Table 4
Financial Moments (Annual)

Table 4: This table reports relevant financial moments for models with either transitory ($\varphi = 0.95$) or permanent ($\varphi = 1.00$) technology shocks and different levels of the elasticity of intertemporal substitution (ψ). The coefficient of relative risk aversion (γ) is 5 across all models. We calibrate the discount factor (β) for each model to match the level of the risk free rate. The volatility of the technology shock (σ_z) is set so that the volatility of consumption growth is the same across models. Capital adjustment costs (ξ) are set so that (if possible) the relative volatility of consumption to output growth is matched. We use annual U.S. data from 1929 to 1998 from the Bureau of Economic Analysis for the empirical moment values. The sample is the same as in Bansal and Yaron (2004).

	LRR I	Model 2	Model 3	Model 4	Model 5	LRR II	
	Transitory Shocks $\varphi = 0.95, \gamma = 5$			Permanent Shocks $\varphi = 1.00, \gamma = 5$			
Statistic	U.S. data 1929-1998	$\psi = 0.05$ $\beta = 1.064$ $\xi = 0.70$ $\sigma_z = 2.59\%$	$\psi = \frac{1}{\gamma}$ $\beta = 1.016$ $\xi = 1.15$ $\sigma_z = 2.59\%$	$\psi = 1.5$ $\beta = 0.998$ $\xi = 6.75$ $\sigma_z = 2.59\%$	$\psi = 0.05$ $\beta = 1.067$ $\xi = \infty$ $\sigma_z = 1.03\%$	$\psi = \frac{1}{\gamma}$ $\beta = 1.012$ $\xi = \infty$ $\sigma_z = 1.20\%$	$\psi = 1.5$ $\beta = 0.998$ $\xi = 18.0$ $\sigma_z = 2.63\%$
<i>The Risk Free Rate</i>							
$E[R_f]$ (%)	0.86	0.85	0.90	1.84	2.93	2.34	0.82
$\sigma[R_f]$ (%)	0.97	4.60	1.40	0.17	0.56	0.41	0.45
<i>The Consumption Claim</i>							
$E[R^A - R_f]$ (%)	n/a	9.68	1.83	0.05	-0.38	-0.15	1.59
$\sigma[R^A - R_f]$ (%)	n/a	29.15	13.23	1.85	4.74	1.06	4.41
$SR[R^A]$	n/a	0.34	0.14	0.03	-0.08	-0.14	0.36
<i>The Dividend Claim</i>							
$E[R^E - R_f]$ (%)	6.33	8.06	1.18	0.04	0.00	0.01	0.24
$\sigma[R^E - R_f]$ (%)	19.42	24.06	8.58	1.59	0.06	0.07	0.66
$SR[R^E]$	0.33	0.34	0.14	0.03	0.08	0.14	0.36

The Dividend Claim. The dividend claim is the claim to the aggregate dividend stream. This claim is unlevered, in contrast to what is the case for the empirical aggregate equity market statistics we report (it is likely that the empirical volatility and risk premium of the unlevered equity return are substantially lower). Again, we see that the volatility of returns is increasing in the capital adjustment costs. For model "LLR I" (the transitory shock case), the volatility of the dividend claim is 24.06% per year. This is too high, especially in the light of these returns being unlevered. Model "LLR II", which has much lower capital adjustment costs, has a volatility of returns to wealth of only 0.66%. This gives an annual equity premium of 0.24%, which is too low to be explained by the lack of leverage.

The two long-run risk models presented here have too high and too low volatility of equity returns, respectively. However, the permanent shock model produces a stochastic discount factor which is in line with the data. Many papers define dividends as a levered claim to the consumption stream, in order to fit the volatility of dividend growth, the high equity return volatility and the equity risk premium. With a leverage factor of about 3 on the consumption claim, the resulting "equity" return premium for the "LLR II" Model would be around 4.5% with a return volatility of about 13%.

But why is it that the dividend claim has so low volatility in the permanent shock model when the consumption claim has a volatility that is an order of magnitude higher and dividends are the residual cash flow? Intuitively, we would expect the dividend claim to be more volatile. The answer lies with the dynamic behavior of dividends.

The production economy model generates dividends endogenously and the endogenous dividend process differs from the endogenous consumption process along important dimensions: While equity dividends are given by $D_t^E = \alpha Y_t - I_t$, dividends to the wealth portfolio are given by $D_t^A = C_t = Y_t - I_t$. Consider a permanent, positive shock to technology. If investors have higher *EIS*, this results in higher investment and higher expected future consumption growth. Both equity dividends as well as dividends to the wealth portfolio now respond less to a positive shock. However, equity dividends are much more sensitive to this effect since $\alpha = 0.34$, and may even decrease in response to a shock, implying a negative correlation between dividend growth and expected consumption growth. So, while the price of the equity claim increases, the current dividend decreases, which dampens the total equity return response to technology shocks. The result is that the equity return volatility, and thus the equity premium, increase less with the *EIS* relative to the total asset return. Figure 4 shows the impulse response of the equity return and dividends to a positive shock to technology in both the long-run risk models.

Figure 4 - Impulse Response 2

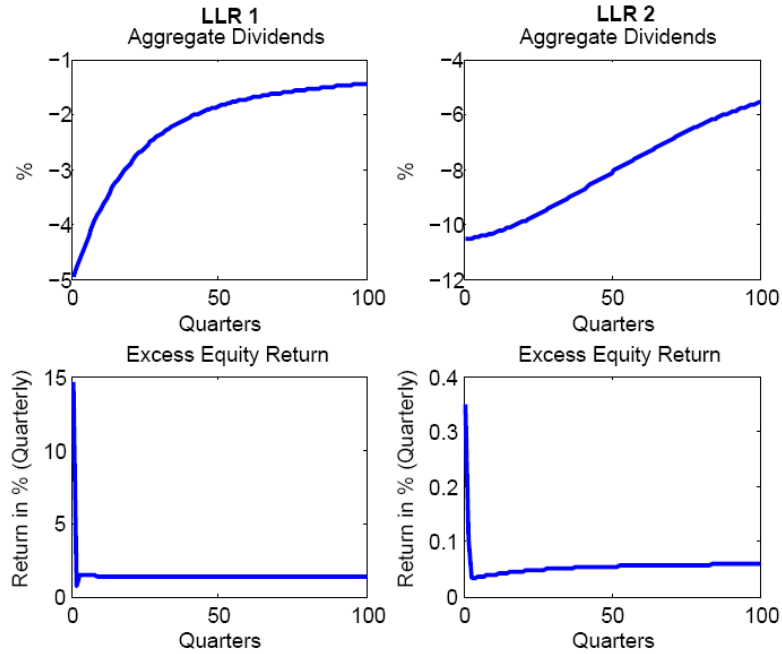


Figure 4: **Impulse-Responses for Dividends and Excess Equity Returns.** The plots show the impulse-responses of dividends and excess equity returns for the LLR I (transitory technology shocks) and the LLR II (permanent technology shocks).

In an exchange economy, it is possible to exploit the fact that the claims to total wealth and equity have different dividend processes (i.e., consumption and dividends), and use this as a degree of freedom to fit the asset pricing moments. Bansal and Yaron (2004), for instance, exogenously specify the dividend process such that expected dividend growth is very sensitive to shocks to expected consumption growth, which makes the equity claim risky and volatile. That way they are able to fit the equity return volatility, and thus the equity premium, with roughly the same (exogenous) consumption process and preference parameters as in the "LLR II" Model. The production economy model, on the other hand, restricts the joint dynamic behavior of aggregate consumption and dividends. Thus, while the general equilibrium framework considered so far in this paper provides a theoretical justification for a consumption process with long-run risk, it imposes constraints on dividends that are unfavorable in terms of matching the volatility of equity returns. We take a closer look at those constraints in the following section.

4.5 Discussion

We have demonstrated that a real business cycle model with capital adjustment costs can generate substantial endogenous long-run risk as an outcome of the optimal consumption-savings decision. The predictability of consumption growth is still quite low, and we will show below that it is hard to detect in small samples of data like the ones we have available. This is an important point, because empirically consumption growth is not highly predictable. The model with permanent technology shocks and a high EIS (LLR II) can be viewed as a justification for the consumption dynamics assumed in Bansal and Yaron (2004), and the unconditional moments of the price of risk and the risk free rate from that model are very similar to Bansal and Yaron (2004) find for their exchange economy model. The models presented here do not, however, generate economically significant time-variation in the price of risk or in the equity risk premium.¹⁵ Further, as can be seen from figures 3 and 4, dividends and consumption are negatively correlated at fairly long time-horizons. In the data they are positively correlated. In the next section, we propose a simple extension that makes dividends pro-cyclical and at the same time increases the equity premium (and return volatility) in the permanent shock model by more than an order of magnitude.

4.5.1 Long-Run Risk Model II: Alternative Specification for Wages

In this section, we argue that a promising avenue for future research is to carefully consider the mechanisms for labor supply and wages within the standard production economy model. So far, we have considered an economy where agents supply a constant amount of labor and where wages are set such that it is optimal for the firm to demand exactly the same amount of labor: wages equal the marginal product of labor. The equilibrium total wages paid are then $W_t = (1 - \alpha)Y_t$. Thus, log wage growth is perfectly correlated with and as volatile as log output growth. In the data, however, wages are only weakly procyclical and less volatile than output. In this section, we specify the wage process so as to match the empirical correlation of wages with output and the relative volatility of wages and output and show that this wage process, which is thus closer to what we observe in the data, allows the model to also generate a process for dividends that is much closer to the data. As a result, the equity premium increases by an order of magnitude.

In the recent labor market search literature, less volatile and less procyclical wages have

¹⁵There is endogenous heteroskedasticity in shocks to both realized and expected consumption growth, but these tend to cancel. We will explain this aspect in more detail in future versions of the paper.

been identified as an important avenue for making operating profits, firm value, and ultimately employment more volatile and more procyclical.¹⁶ We essentially propose the same in order to alleviate the equity premium puzzle in our model. Instead of assuming that labor is paid its marginal product, we postulate a different wage process and calibrate that process to the data. In the presence of labor market frictions, for instance a search and matching friction, there is no reason to assume that labor is paid its marginal product. The search and matching literature assumes instead that wages are an outcome of a bargaining process between firm and worker. The wage process we propose is similar to the sticky wage rule Hall (2005) proposes. The consequence of less volatile wages are more volatile and more procyclical firm operating profits and firm dividends compared to the original model. This in turn leads to more volatile firm values and equity returns and to a higher equity risk premium. One way of viewing this is that we increase the operating leverage of the firm by introducing a less volatile and less procyclical "fixed-cost-component".

We specify the following wage process:

$$W_t^{adj} = \omega_0 (\omega_1 Y_t + (1 - \omega_1) Y_{t-1}), \quad (20)$$

so that wages are a weighted average current and last period output. We calibrate this process to U.S. data from 1952 to 2004.¹⁷

Table 5 reports asset pricing moments for the model with permanent technology shocks and a high EIS (LLR II). We report both the original equity return, that is the return of a claim on the original dividend process, as well as the adjusted equity return, that is a claim on the new dividend process ($D_t^{adj} = Y_t - W_t^{adj} - I_t$). Note that the return on investment is no longer equal to the return on the equity claim in this case. Therefore, we solve numerically for the equity claim using the value function iteration algorithm.

From Table 5 we can see that the standard production economy model has the potential to match both the level as well as the volatility of the equity premium. With our simple adjustment of the process for wages, the premium of the unlevered equity return increases

¹⁶In that literature, the counterfactually low volatility of employment, induced by too low volatility of firm values, has been a long-standing puzzle, dubbed the "Shimer puzzle". For accounts of (and solutions for) the lack of movement in employment within the standard Mortensen and Pissarides (1994) labor market matching framework, see for example Den Haan, Ramey, Watson (2000), Hornstein, Krusell, Violante (2005), Shimer (2005), and Den Haan and Kaltenbrunner (2006). Making wages "stickier" has been originally promoted as an important contribution to the resolution of the Shimer puzzle by Hall (2005).

¹⁷We set $\omega_1 = 0.28$ in order to match $corr(\Delta w, \Delta y) = 0.38$. As a result, $\sigma(\Delta w) / \sigma(\Delta y) = 0.78$, which turns out to be close to its empirical counter-part.

Table 5

Adjusted Wage Process — Asset Pricing Moments

Table 5: This table reports asset pricing moments for the Long-Run Risk II - Model, which has permanent technology shocks, an elasticity of intertemporal substitution (ψ) of 1.5, and relative risk aversion (γ) of 5. We report both the original equity return, that is the return of a claim on the original dividend process, as well as the adjusted equity return, that is a claim on the adjusted dividend process. The data are taken from Bansal and Yaron (2004) who use U.S. financial markets data from 1929 to 1998. All values reported in the table are annual.

Statistic	Data	Long-Run Risk II
$\sigma [\Delta c]$ (%)	2.72	2.72
$\sigma [\Delta c] / \sigma [\Delta y]$	0.52	0.52
$\sigma [M] / E [M]$	<i>n/a</i>	0.36
$E [R_f]$ (%)	0.86	0.82
$\sigma [R_f]$ (%)	0.97	0.45
$SR[R^E]$	0.33	0.36
$E[Rr^E - R_f]$ (%)	6.33	0.24
$\sigma[R^E - R_f]$ (%)	19.42	0.66
$SR[R^{Eadj}]$	0.33	0.36
$E[R^{Eadj} - R_f]$ (%)	6.33	1.46
$\sigma[R^{Eadj} - R_f]$ (%)	19.42	4.06
$\sigma[\Delta d^{adj}]$ (%)	11.49	11.52
$corr [\Delta d^{adj}, \Delta c]$	0.55	0.47
Financial Leverage		
	<i>Data: ($D/V \approx 0.33$)</i>	<i>Model: $D/V = 0.33$</i>
$E[R^{Eadj} - R_f]$ (%)	6.33	2.03
$\sigma[R^{Eadj} - R_f]$ (%)	19.42	6.15

from 0.24% to 1.46%. Because the equity return from the data is the return on a levered equity claim we add financial leverage to our model (the debt is assumed risk free and specified as in Jermann, 1998). Now the model is able to generate an equity premium of 2.03% with an equity return volatility of 6.15%. The model matches the volatility of dividend growth and the correlation between dividend growth and consumption growth (taken from Bansal and Yaron, 2004). Given that we calibrate the wage process and *not* the process for dividends, the model generates a quite realistic dividend process as measured by its volatility and correlation with consumption.

We conclude that a wage process closer to what we observe in the data yields a dividend process, and as a result equity returns, substantially closer to what we find in the data. An interesting avenue for future research is to endogenize labor and hours worked in the model presented in this paper.¹⁸

4.5.2 Predictability

The models presented so far do not generate economically significant time-variation in the equity risk premium (see appendix). We are currently working on extensions of the model where the labor supply is time-varying in part to attempt to address this issue.

5 Empirical Tests: Expected Consumption Growth and the Cross-Section of Stock Returns

We test key predictions of the model for the time series of technology (total factor productivity) and consumption growth, as well as for the cross-section of stock returns. In particular, we test whether proxies for expected consumption growth suggested by our model actually forecast long-horizon consumption growth or not, whether shocks to expected consumption growth are a priced risk factor or not, and whether the price of risk is positive or negative.

In a recent paper, Bansal, Kiku, and Yaron (2006) test an exchange economy version of the model in this paper and show that consumption growth is indeed predictable using forecasting variables such as lagged consumption growth, the default spread, and the market price-dividend ratio. Furthermore, they show, using the cross-section of stock returns, that

¹⁸Kaltenbrunner (2006) incorporates a search and matching model into standard production economy models with habit preferences in order to jointly explain macroeconomic time series, including labor market series, and asset prices.

shocks to expected consumption growth are indeed a positively priced risk factor. We therefore confine our empirical analysis to test restrictions that are particular to the production economy. We consider an instrument Bansal, Kiku, and Yaron do not use and which is related to the level of technology - the driving process of the production economy model.

The consumption data and data on Total Factor Productivity (TFP; the equivalent to "technology" in our model) are obtained from the Bureau of Economic Analysis and the Bureau of Labor Statistics, respectively. The return data are from Kenneth French's homepage.

5.1 Expected Consumption Growth

As highlighted by Harvey and Shepard (1990), Bansal and Yaron (2004) and Hansen, Heaton and Li (2005), amongst others, it is difficult to estimate long-run consumption growth dynamics from the relatively short samples of data we have available. In our model, slow-moving expected consumption growth dynamics arise due to endogenous consumption smoothing, and the production economy model therefore identifies observable proxies for the otherwise unobservable expected consumption growth rate. In particular, the ratio of the level of technology to the level of consumption forecasts future consumption growth with a positive sign when technology shocks are permanent. This intuition is confirmed in Figure 5, which shows the impulse-response of consumption to a one standard deviation permanent shock to technology (total factor productivity) for high and low levels of the *EIS*. Investors respond to a permanent technology shock by increasing investment in order to build up higher levels of capital. Thus, while technology immediately adjusts to its new permanent level, consumption only slowly grows to a permanently higher level as capital needs to be built up to support the new steady state consumption level.

Define:

$$zc_t \equiv \ln \left(\frac{Z_t}{C_t} \right). \quad (21)$$

If zc_t is high in the permanent technology shock case, consumption is expected to increase towards a new steady-state level. Our model thus implies that zc_t is a good instrument for the expected consumption growth rate and, in the permanent shock case, that zc_t should predict future consumption growth with a positive sign. For a transitory technology shock, the consumption response can be hump-shaped. However, the long-run consumption response is in this case always negative. Thus, with transitory technology shocks, the ratio zc_t predicts long-horizon consumption growth with a negative sign.¹⁹ Empirically, however, we find

¹⁹We have confirmed this using data simulated from the LLR I Model (with transitory technology shocks),

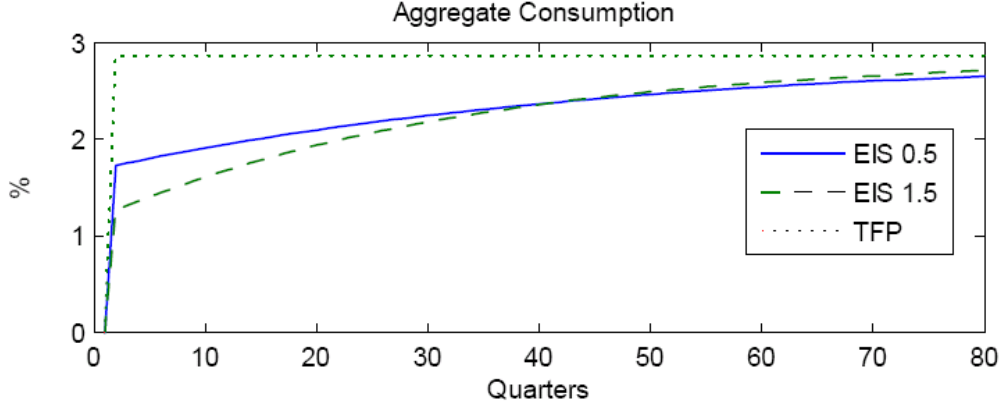


Figure 5: **Impulse-Response of Consumption** Impulse responses of consumption to a one standard deviation positive and permanent shock to technology for different levels of the EIS. The impulse-responses are for a model with EIS = 0.5 and the LLR II Model (EIS = 1.5), respectively.

support for the permanent technology shock case, and in the following we discuss this case in more detail.

In our model, z_{c_t} is stationary even though both Z_t and C_t are non-stationary. In particular, since the production technology is specified as:

$$Y_t = Z_t^{1-\alpha} K_t^\alpha N_t^{1-\alpha}, \quad (22)$$

all endogenous variables in the economy evolve around the (stochastic) trend Z_t (see Appendix 8.4). We get data on Z_t (TFP) from the Bureau of Labor Statistics (BLS). The BLS computes TFP as follows. First it collects data on Y_t (output), on K_t (capital input), and on N_t (labor input). Then the BLS estimates a value for the parameter α and computes TFP as the Solow residual:

$$\ln \tilde{Z}_t = \ln Y_t - \alpha \ln K_t - (1 - \alpha) \ln N_t. \quad (23)$$

Note that the BLS specifies the following production technology:

$$Y_t = \tilde{Z}_t K_t^\alpha N_t^{1-\alpha}. \quad (24)$$

It follows that we need to normalize:

$$Z_t = \tilde{Z}_t^{1/(1-\alpha)}. \quad (25)$$

but the results are not reported for brevity.

We take as the value for α the value we use in our model ($\alpha = 0.34$). We check our results for robustness by assuming different values for $\alpha \in [0.30, 0.40]$, and find that our results are robust with respect to the choice of α .

The model with permanent technology shocks (LLR II), suggests the following forecasting relationship:

$$\Delta c_{t,t+j} = \alpha + \beta zc_t + \varepsilon_{t,t+j}, \quad (26)$$

where $\beta > 0$. In the model the relation is not exactly linear, but when simulating data from our model (LLR II) we find that zc_t accounts for more than 99% of the variation in expected consumption growth in a linear regression. We test this forecasting relationship both on data from 1948 to 2005 and on data generated by the permanent technology shock version of our model (LLR II). In particular, in Table 6 we report results from forecasting regressions of annual log nondurable- and services consumption growth on the lagged log TFP to consumption ratio, our measure of expected consumption growth.

Panel A shows forecasting regressions corresponding to (26) using simulated data from our model. The regression coefficient is increasing with the horizon up to 7 years. The forecasting regression coefficients are found by simulating 10,000 samples of length 58 years, running the regression on each sample, and computing the average regression coefficient. The sample errors are the sample standard deviation of each β -estimate. Interestingly, the regression coefficient is not significant in any of the regressions using data simulated from the model. The variation in expected consumption growth is too slow-moving for the regressions to on average uncover the forecasting relationship over the relatively short sample period. Note, however, that there is a small-sample bias in these regressions. Shocks to consumption growth and the *zc-ratio* are positively correlated, since technology moves more than consumption in response to a shock. Therefore, the regression coefficient is biased towards zero (see Stambaugh, 1999) in the above regressions.

Panel B shows the results from the forecasting regressions using real data. Here both the regression coefficients and the R^2 's are increasing with the forecasting horizon. The coefficients are significant at the 10% level for all regressions using Hodrick (1992) standard errors, which have relatively good small sample properties for overlapping regressions. The coefficients are overall lower than those estimated using simulated data. This could be because there is less variation in expected consumption growth in the data or because the

Table 6

Estimating Expected Consumption Growth

Table 6: This table reports forecasting regressions of annual log nondurable- and services consumption growth on a lagged measure of expected consumption growth, the log TFP to Consumption ratio. The consumption and TFP data are from the Bureau of Economic Analysis and the Bureau of Labor Statistics respectively. We use annual data from 1948 to 2005, resulting in $58 - j$ observations for a regression with a j year forecasting horizon: Multi-year forecasting regressions are overlapping at an annual frequency. The standard error estimates (in parenthesis) are corrected for heteroskedasticity and overlapping observations using Hodrick (1992) standard errors. Results for the model are based on 10,000 replications of sample size 58×4 each. Numbers in bold indicate significance at the 5% level or more in a two-tailed t-test, while an asterisk indicates significance at the 10% level.

Regression: $\Delta c_{t,t+j} = \alpha + \beta z c_t + \varepsilon_{t,t+j}$

Panel A: Model implied (j denotes forecasting horizon in years)

j	1	2	3	4	5	7	10
β	0.077 (0.051)	0.135 (0.101)	0.178 (0.151)	0.208 (0.201)	0.226 (0.251)	0.233 (0.345)	0.192 (0.468)
R^2_{adj}	9.2% (0.092)	10.2% (0.118)	10.9% (0.128)	10.8% (0.131)	10.3% (0.131)	9.3% (0.128)	8.3% (0.124)

Panel B: Historical estimates (j denotes forecasting horizon in years)

j	1	2	3	4	5	7	10
β	0.021* (0.012)	0.041* (0.024)	0.060* (0.036)	0.084* (0.048)	0.107* (0.060)	0.147* (0.085)	0.233* (0.127)
R^2_{adj}	3.3%	5.5%	7.6%	11.1%	13.8%	17.6%	30.0%

empirical *zc-ratio* is measured with noise.²⁰

We conclude that the log TFP to consumption ratio, a measure of expected consumption growth implied by our theoretical model, forecasts future consumption growth with a positive sign and that this supports a model with permanent technology shocks. The amount of variation in expected consumption growth in the data, as measured by these regressions, is similar in magnitude to that implied by the model with permanent technology shocks and $\gamma = 5$ and $\psi = 1.5$ (LLR II).

5.2 The Cross-Section of Stock Returns

The model in this paper implies that the shock to expected consumption growth is a priced risk factor as long as $\gamma \neq \frac{1}{\psi}$, i.e. as long as agents care about the temporal resolution of risk. The cross-section of stock returns can tell us both whether shocks to expected consumption growth are a priced risk factor and whether the price of risk on this factor is positive or negative, which in turn depends on whether the relative risk aversion of the representative agent is smaller or larger than the reciprocal of the elasticity of substitution. To relate the consumption dynamics directly to the stochastic discount factor, we assume that the dynamic behavior of consumption growth generated by the model follows:

$$\Delta c_{t+1} = \mu + x_t + \eta_{t+1}, \quad (27)$$

$$x_{t+1} = \rho x_t + e_{t+1}, \quad (28)$$

$$\sigma_{\eta,e} = \text{corr}(e_{t+1}, \eta_{t+1}). \quad (29)$$

We verify in the appendix that this parsimonious specification captures the true behavior of consumption growth from our model reasonably well. The effect of transitory versus permanent technology shocks is reflected in the correlation between the innovations to realized and expected consumption growth: In the case of transitory technology shocks, the correlation is negative, while in the case of permanent technology shocks the correlation is positive.

Given the consumption dynamics in (27) and log-linearizing the return on the wealth portfolio around the steady state ratio of wealth to aggregate consumption, the stochastic

²⁰The R^2 's are higher for the long-horizon regressions than predicted by the model, although Valkanov (2003) cautions that R^2 's are badly behaved in small samples where the fraction of overlapping observations relative to the total sample length is large. Thus, the sample R^2 's are likely to overstate the true R^2 's.

discount factor can be written:

$$m_{t+1} \approx a - b_1 \Delta c_{t+1} - b_2 e_{t+1} - b_3 x_t, \quad (30)$$

where Δc_{t+1} denotes realized consumption growth, x_t is the current level of expected consumption growth, e_{t+1} is the shock to expected consumption growth, and $b_1 = \gamma$, $b_2 = (1 - \theta) A_1 \kappa_1$, $b_3 = (\theta - 1) A_1 (1 - \kappa_1 \rho)$ (see the appendix for a detailed derivation and definitions of the constants $A_1, \kappa_1 > 0$). If $\gamma > \frac{1}{\psi}$, the coefficients $b_1, b_2 > 0$ and $b_3 < 0$. We will test these restrictions using the cross-section of stock returns.

We use the log ratio of TFP to consumption, $z c_t$, to obtain measures of x_t and e_{t+1} from the following regressions:

$$\Delta c_{t+1} = \hat{k}_0 + \hat{k}_1 z c_t + \hat{\eta}_{t+1}, \quad (31)$$

$$\hat{x}_t = \hat{k}_1 (z c_t - E_T [z c_t]), \quad (32)$$

$$z c_{t+1} = \hat{k}_3 + \hat{k}_4 z c_t + \hat{\nu}_{t+1}, \quad (33)$$

$$\hat{e}_{t+1} = \hat{k}_1 [z c_{t+1} - \hat{k}_3 - \hat{k}_4 z c_t], \quad (34)$$

where $E_T [\cdot]$ denotes the sample mean and \hat{k}_i is an OLS regression coefficient. The permanent shock model predicts that shocks to realized ($\hat{\eta}$) and expected (\hat{e}) consumption are positively correlated, which we confirm is the case in our sample.

By applying a standard log-linear approximation of the stochastic discount factor (see appendix), we arrive at the linear factor model:

$$\begin{aligned} E [R_{i,t+1} - R_{0,t+1}] &= b_1 Cov (\Delta c_{t+1}, R_{i,t+1} - R_{0,t+1}) + b_2 Cov (e_{t+1}, R_{i,t+1} - R_{0,t+1}) \\ &\quad + b_3 Cov (x_t, R_{i,t+1} - R_{0,t+1}). \end{aligned} \quad (35)$$

The standard consumption-based asset pricing model with power utility implies that $b_1 = \gamma = \frac{1}{\psi}$, while $b_2 = b_3 = 0$. As noted above, if $\gamma > \frac{1}{\psi}$, however, $b_2 > 0$ and $b_3 < 0$.

Because TFP data are only available on an annual basis from the Bureau of Economic Analysis, we use annual excess returns on the 25 Fama-French portfolios as test assets. The sample consists of 57 observations from 1948 to 2005. Table 7 displays results for the benchmark Consumption CAPM ($b_2 = b_3 = 0$) and the three-factor model of this paper.

Table 7 displays the sign of the estimated quantities with p-values in parentheses. The factor loading on realized consumption growth risk is insignificant for the standard Con-

Table 7
The Price of Long-Run Risk from Cross-Sectional Regressions

Table 7: This table reports the estimated loadings on the factors of the Consumption CAPM and the long-run risk production-based model developed in this paper (Prod.CAPM). Test assets are the 25 Fama-French portfolios sorted by size and book-to-market equity ratios. We use returns in excess of a risk-free rate (U.S. t-bill). All variables are annual. There are 57 observations from 1949 - 2005. Estimation is by two-pass regression, where the standard errors are corrected for generated regressors (Shanken, 1992). P-values are reported for each variable, where the null hypothesis is that the estimate is zero. Numbers in bold indicate significance at the 10% level or more in a two-tailed t-test.

Tests of Factor Significance: $m_{t+1} = a - b_1\Delta c_{t+1} - b_2e_{t+1} - b_3x_t$		
Factor loading b	<i>Cons.CAPM</i>	<i>Prod.CAPM</i>
Realized Cons. Growth (b_1) (p-value)	> 0 (0.24)	< 0 (0.82)
Shock to Exp. Cons. Growth (b_2) (p-value)		$> \mathbf{0}$ (0.09)
Expected Cons. Growth (b_3) (p-value)		< 0 (0.11)
Joint test: $b_2 = b_3 = 0$ (p-value)		<i>reject</i> (0.06)
R_{adj}^2	17.4%	46.6%

sumption CAPM model and the adjusted cross-sectional R^2 is 17.4%. The three-factor model including measures of the level and the shock to expected consumption growth increases the adjusted R^2 to 46.6%. The factor loading on the shock to expected consumption growth carries a positive sign and is significant at the 10% level. The sign on the coefficient on the measure of expected consumption growth (b_3) is negative as predicted, but not quite significant. The power utility benchmark implies that $b_2 = b_3 = 0$. A test of this joint hypothesis yields a p-value of 0.06. Thus, the statistical evidence is not very strong, but significant at the 10% level. On the other hand, we are relying on a noisy proxy and the sample is fairly small.

We conclude that the linear three-factor model derived from our theoretical model outperforms the benchmark Consumption CAPM. We reject the null hypothesis that long-run risk is not important relative to the standard Consumption CAPM for the cross-section of stock returns. The signs on factor loadings b_2 and b_3 are consistent with a model where agents prefer early resolution of uncertainty ($\gamma > \frac{1}{\psi}$).

6 Conclusion

We analyze a standard stochastic growth model with capital adjustment costs and Epstein-Zin preferences. We show that long-run risk arises endogenously as a consequence of consumption smoothing, both when log technology follows a random walk or an AR(1).

When the coefficient of relative risk aversion is greater than the reciprocal of the elasticity of intertemporal substitution, agents prefer early resolution of uncertainty and dislike shocks to future economic growth prospects. Unlike in the case of power utility, shocks to *expected* consumption growth now also appear as a risk factor. The presence of long-run risk in this case decreases the market price of risk if technology shocks are transitory, while it increases the market price of risk if technology shocks are permanent. With permanent technology shocks, the model matches the level and volatility of the risk free rate, the unconditional Sharpe ratio of equities and the low volatility of consumption growth. The model achieves this with a low level of relative risk aversion, unlike habit formation models where typical implementations also generate too much volatility in the risk free rate. The volatility of equity returns, however, is an order of magnitude too low. We propose a simple extension of the model where wages are calibrated to be less volatile than output and only slightly pro-cyclical, as in the data. This increases the equity premium significantly and suggests that richer dynamics in the labor supply and wage determination is an interesting avenue

for future research.

If agents instead prefer late resolution of uncertainty, the opposite pattern emerges - long-run risk increases the market price of risk if technology shocks are transitory, while it decreases the market price of risk if technology shocks are permanent. In this case, long-run risks arise in the case where agents have *low* elasticity of intertemporal substitution. However, a low elasticity of intertemporal substitution also leads to too high volatility of the risk free rate. We find that the elasticity of intertemporal substitution, which strongly affects the dynamics of the macroeconomic variables, also strongly affects the price of risk and the Sharpe ratio of equity in all our calibrations of the model. Thus, there is a tight link between quantity dynamics and asset prices in our implementation of the standard stochastic growth model.

The model provides a theoretical justification for a long-run risk component in aggregate consumption growth. This result is of particular interest since it is very difficult to empirically distinguish a small predictable component of consumption growth from i.i.d. consumption growth given the short sample of data we have available. In particular, the production economy model identifies the ratio of technology to consumption as a proxy for the otherwise hard to estimate expected consumption growth. We test this link in the time-series of consumption growth and in the cross-section of stock returns. We find support for both tests. In particular, the production-based CAPM outperforms the standard CCAPM in a cross-sectional test. The results from the time-series and the cross-sectional analysis are consistent with a model where technology shocks are permanent and agents have a preference for early resolution of uncertainty.

7 References

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8 Appendix

8.1 Model Solution

The Return to Investment and the Firm's Problem The firm maximizes firm value. Let $M_{t,t+1}$ denote the stochastic discount factor. The firm's problem is then:

$$\max_{\{I_t, K_{t+1}, H_t\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} M_{0,t} \left\{ q_t \left(K_{t+1} - (1 - \delta) K_t - \phi \left(\frac{I_t}{K_t} \right) K_t \right) \right\} \right], \quad (36)$$

where q_t denotes the shadow price of the capital accumulation constraint, equivalent to marginal q : The expected present value of one marginal unit of capital. Maximizing over labor we obtain $(1 - \alpha) Z_t^{1-\alpha} K_t^\alpha H_t^{-\alpha} = W_t$ and $H_t = (1 - \alpha)^{\frac{1}{\alpha}} Z_t^{\frac{1}{\alpha}-1} W_t^{-\frac{1}{\alpha}} K_t$. In other words, we assume an exogenous wage process such that it is optimal for the firm to always hire at full capacity ($H_t = 1$), which is the same amount of labor as the representative agent is assumed to supply. In this case, total wages $W_t H_t = W_t = (1 - \alpha) Y_t$, so wages are pro-cyclical and have the same growth rate volatility as total output. The operating profit function of the firm follows as:

$$\begin{aligned} \Pi(K_t, Z_t; W_t) &= Z_t^{1-\alpha} \left[(1 - \alpha)^{\frac{1}{\alpha}} Z_t^{\frac{1}{\alpha}-1} W_t^{-\frac{1}{\alpha}} K_t \right]^{1-\alpha} K_t^\alpha - W_t (1 - \alpha)^{\frac{1}{\alpha}} Z_t^{\frac{1}{\alpha}-1} W_t^{-\frac{1}{\alpha}} K_t \\ &= Z_t^{1-\alpha} \left[(1 - \alpha)^{\frac{1}{\alpha}} Z_t^{\frac{1}{\alpha}-1} W_t^{-\frac{1}{\alpha}} \right]^{1-\alpha} K_t - (1 - \alpha)^{\frac{1}{\alpha}} Z_t^{\frac{1}{\alpha}-1} W_t^{1-\frac{1}{\alpha}} K_t \end{aligned} \quad (37)$$

$$= \left((1 - \alpha)^{\frac{1}{\alpha}-1} Z_t^{\frac{1}{\alpha}-1} W_t^{1-\frac{1}{\alpha}} - (1 - \alpha)^{\frac{1}{\alpha}} Z_t^{\frac{1}{\alpha}-1} W_t^{1-\frac{1}{\alpha}} \right) K_t \quad (38)$$

$$= \left(\alpha (1 - \alpha)^{\frac{1}{\alpha}-1} Z_t^{\frac{1}{\alpha}-1} W_t^{1-\frac{1}{\alpha}} \right) K_t. \quad (39)$$

The operating profit function of the firm is thus linearly homogenous in capital. Substituting out equilibrium wages we obtain $\Pi(K_t, Z_t; W_t) = \alpha Y_t$. We re-state the firm's problem:

$$\max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} M_{0,t} \left\{ \Pi(\cdot) - I_t - q_t \left(K_{t+1} - (1 - \delta) K_t - \phi \left(\frac{I_t}{K_t} \right) K_t \right) \right\} \right]. \quad (40)$$

Each period in time the firm decides how much to invest, taking marginal q as given. The first order conditions with respect to I_t and K_{t+1} are immediate:

$$0 = -1 + q_t \phi' \left(\frac{I_t}{K_t} \right), \quad (41)$$

and

$$0 = -q_t + E_t \left[M_{t+1} \left\{ +q_{t+1} \left((1 - \delta) - \phi' \left(\frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} + \phi \left(\frac{I_{t+1}}{K_{t+1}} \right) \right) \right\} \right]. \quad (42)$$

Substituting out q_t and q_{t+1} in (42) yields:

$$\frac{1}{\phi' \left(\frac{I_t}{K_t} \right)} = E_t \left[M_{t+1} \left\{ \Pi_K(\cdot) + \frac{(1 - \delta) - \phi' \left(\frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} + \phi \left(\frac{I_{t+1}}{K_{t+1}} \right)}{\phi' \left(\frac{I_{t+1}}{K_{t+1}} \right)} \right\} \right], \quad (43)$$

$$1 = E_t \left[M_{t+1} \left\{ \phi' \left(\frac{I_t}{K_t} \right) \left(\Pi_K(\cdot) + \frac{1 - \delta + \phi \left(\frac{I_{t+1}}{K_{t+1}} \right)}{\phi' \left(\frac{I_{t+1}}{K_{t+1}} \right)} - \frac{I_{t+1}}{K_{t+1}} \right) \right\} \right], \quad (44)$$

$$1 = E_t [M_{t+1} R_{t+1}^I]. \quad (45)$$

Equation (45) is the familiar law of one price, with the firm's return to investment:

$$R_{t+1}^I = \phi' \left(\frac{I_t}{K_t} \right) \left(\Pi_K(K_{t+1}, Z_{t+1}; W_{t+1}) + \frac{1 - \delta + \phi \left(\frac{I_{t+1}}{K_{t+1}} \right)}{\phi' \left(\frac{I_{t+1}}{K_{t+1}} \right)} - \frac{I_{t+1}}{K_{t+1}} \right). \quad (46)$$

8.2 Risk and the Dynamic Behavior of Consumption

Epstein-Zin preferences have been used with increasing success in the asset pricing literature over the last years (e.g., Bansal and Yaron, 2004, Hansen, Heaton and Li, 2005, Malloy, Moskowitz and Vissing-Jorgensen, 2005, Yogo, 2006). This is both due to their recursive nature, which allows time-varying growth rates to increase the volatility of the stochastic discount factor through the return on the wealth portfolio, as well as the fact that these

preferences allow a convenient separation of the elasticity of intertemporal substitution from the coefficient of relative risk aversion.

Departing from time-separable power utility preferences with $\gamma = \frac{1}{\psi}$ means agents care about the temporal distribution of risk. This is a key assumption of our analysis, because it is precisely this departure from the classic preference structure that renders time-varying expected consumption growth rates induced by optimal consumption smoothing behavior a priced risk factor in the economy.

8.2.1 Early Resolution of Uncertainty and Aversion to Time-Varying Growth Rates

To gain some intuition for why a preference for early resolution of uncertainty implies aversion to time-varying growth rates, we revisit an example put forward in Duffie and Epstein (1992). Consider a world where each period of time consumption can be either high or low. Next, the consumer is given a choice between two consumption gambles, A and B . Gamble A entails eating $C_0 \equiv \frac{1}{2}C^H + \frac{1}{2}C^L$ today, where C^H is a high consumption level and C^L is a low consumption level. Tomorrow you flip a fair coin. If the coin comes up heads, you will get C^H each period forever. If the coin comes up tails, you will get C^L each period forever. Gamble B entails eating C_0 today, and then flip a fair coin *each* subsequent period t . If the coin comes up heads at time t , you get C^H at time t , and if it comes out tails, you get C^L at time t . Thus, in the first case uncertainty about future consumption is resolved early, while in the second case uncertainty is resolved gradually (late). If $\gamma = \frac{1}{\psi}$ (power utility), the consumer is indifferent with respect to the timing of the resolution of uncertainty and thus indifferent between the two gambles. However, an agent who prefers early resolution of uncertainty (i.e., she likes to plan), prefers gamble A .

We can now also phrase this discussion in terms of *growth rates*. From this perspective, gamble A has constant expected consumption growth, while gamble B has a mean-reverting process for expected consumption growth. Thus, a preference for early resolution of uncertainty translates into an aversion of time-varying expected consumption growth.

Another, more mechanical, way to see this is by directly looking at the stochastic discount factor. It is well known, e.g. Rubinstein (1976), that the stochastic discount factor, M_{t+1} , is the ratio of the representative agent's marginal utility between today and tomorrow:

$M_{t+1} = \frac{U'(C_{t+1})}{U'(C_t)}$. Using a recursive argument, Epstein and Zin (1989) show that:

$$\ln M_{t+1} \equiv m_{t+1} = \theta \ln \beta - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) r_{a,t+1}, \quad (47)$$

where $\Delta c_{t+1} \equiv \ln \frac{C_{t+1}}{C_t}$ and $r_{a,t+1} \equiv \ln \frac{C_{t+1} + A_{t+1}}{A_t}$ is the return on the total wealth portfolio with A_t denoting total wealth at time t .²¹ If $\gamma = \frac{1}{\psi}$, $\theta = \frac{1-\gamma}{1-1/\psi} = 1$, and the stochastic discount factor collapses to the familiar power utility case. However, if the agent prefers early resolution of uncertainty, the return on the wealth portfolio appears as a risk factor. More time-variation in expected consumption growth (the expected "dividends" on the total wealth portfolio) induces higher volatility of asset returns, in turn resulting in a more volatile stochastic discount factor and thus a higher price of risk in the economy.²²

The effect on the equity premium can be understood by considering a log-linear approximation (see Campbell, 1999) of returns and the pricing kernel, yielding the following expressions for the risk free rate and the equity premium:

$$r_{f,t+1} \approx -\log \beta + \frac{1}{\psi} E_t [\Delta c_{t+1}] - \frac{\theta}{2\psi^2} \sigma_{t,c}^2 + \frac{(\theta - 1)}{2} \sigma_{t,r^A}^2, \quad (48)$$

$$E_t [r_{t+1}^E] - r_{f,t+1} \approx \frac{\theta}{\psi} \sigma_{t,r^E c} + (1 - \theta) \sigma_{t,r^E r^A} - \frac{\sigma_{t,r^E}^2}{2}, \quad (49)$$

where $E_t [\Delta c_{t+1}]$ is expected log consumption growth, $\sigma_{t,c}$, σ_{t,r^A} , σ_{t,r^E} , are the conditional standard deviations of log consumption growth, the log return on the total wealth portfolio, and the log equity return, and $\sigma_{t,r^E c}$ and $\sigma_{t,r^E r^A}$ are the conditional covariances of the log equity return with log consumption growth and the log return on the total wealth portfolio respectively. We can see how the level of the equity premium depends directly on the covariance of equity returns with returns on the wealth portfolio.

8.2.2 Predictability

The models presented in this paper do not yield economically significant time-variation in the equity premium. The technology shocks are homoskedastic and risk preferences are constant, so any time-variation in the price of risk and/or equity premium must come from

²¹Note that our representative household's total wealth portfolio is composed of the present value of future labor income in addition to the value of the firm.

²²This assumes that the correlation between the return on the wealth portfolio and consumption growth is non-negative, which it is for all parameter values we consider in this paper (and many more).

Figure 6 - Conditional Moments

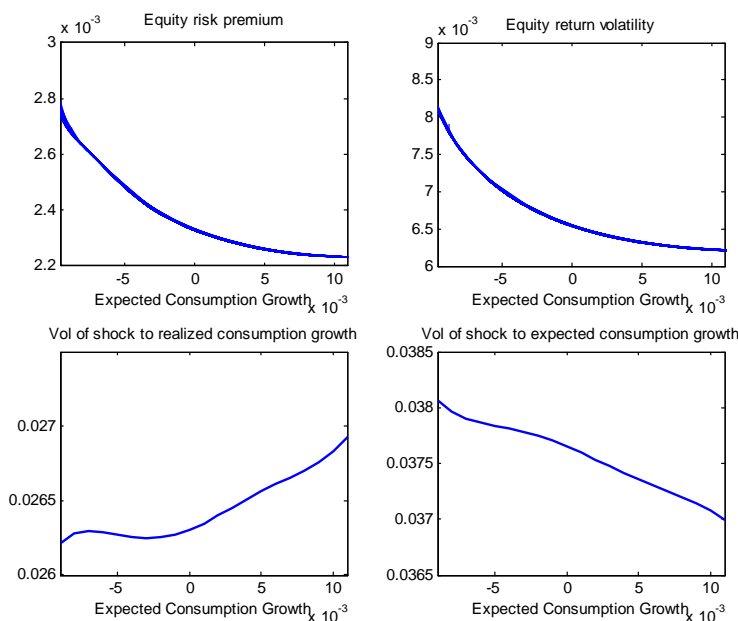


Figure 6: **Conditional Moments.** The plots show the conditional equity risk premium, return volatility, volatility of shocks to realized and expected consumption growth plotted against the conditional level of expected consumption growth. The latter is inversely related to the level of capital relative to the level of technology.

endogenous heteroskedasticity in the consumption (and/or dividend) process. Figure 6 shows the equity risk premium for the model with permanent technology shocks (LLR II) plotted against the conditional expected consumption growth. When capital is low, relative to the level of technology, expected consumption growth is high in the permanent shock model as the marginal productivity of capital is high and agents therefore invest. Thus, these are good times. When expected consumption growth is low, investment is low, and we associated this with a recession. The figure shows that the equity risk premium is higher in recessions. However, the magnitude of the time-variation is too small to yield predictability regressions with the same R^2 's as in the data (not reported). There is some endogenous heteroskedasticity in shocks to both realized and expected consumption growth, but the two go in opposite directions. The net effect is a price of risk that is almost constant.

8.2.3 Technology and Risk Aversion

Standard production technologies do not allow agents to hedge the technology shock. Agents must in the aggregate hold the claim to the firm's dividends. Therefore, the only action available to agents at time t in terms of hedging the shock at time $t + 1$, is to increase savings in order to increase wealth for time $t + 1$. The shock will still hit the agents at time $t + 1$ though, no matter what. Wealth levels may be higher if a bad realization of the technology shock hits the agents, but wealth is also higher if a good realization of the technology shock occurs. The *difference* between the agents' utility for a good realization of the technology shock in period $t + 1$ relative to their utility for a bad realization of the shock is thus (almost) unaffected. However, it is this utility *difference* the agents care about in terms of their risk aversion. Now, because the agents' utility function is concave, this is not quite true. A higher wealth level in both states of the world does decrease the difference between utility levels. Agents thus respond by building up what is referred to as "buffer-stock-savings". This is, however, a second-order effect. As a result, the dynamic behavior of consumption growth is largely unaffected by changing agents' coefficient of relative risk aversion. The fundamental consumption risk in the economy remains therefore (almost) the same when we increase risk aversion (γ) while holding the *EIS* (ψ) constant. Asset prices, of course, respond as usual to higher levels of risk aversion.

Table 8 confirms this result for calibrations with both a coefficient of relative risk aversion (γ) of 5, as well as versions of the models with a higher level of risk aversion ($\gamma = 25$).

8.3 Accuracy of the Approximation of the Endogenous Consumption Process

In Section 8.4 we propose the following approximation for the dynamics of the endogenous process for consumption:

$$\Delta c_{t+1} = \mu + x_t + \sigma_\eta \eta_{t+1}, \quad (50)$$

$$x_{t+1} = \rho x_t + \sigma_e e_{t+1}, \quad (51)$$

$$\sigma_{\eta,e} = \text{corr}(\eta_{t+1}, e_{t+1}). \quad (52)$$

Here Δc_{t+1} is log realized consumption growth, x_t is the time-varying component of expected consumption growth, and η_t, e_t are zero mean, unit variance, and normally distributed disturbance terms with correlation $\sigma_{\eta,e}$. This functional form for log consumption growth is

Table 8

The Effect of Risk Aversion on Macroeconomic Time Series

Table 8: This table reports relevant macroeconomic moments and consumption dynamics for the long-run risk models (LLR I and LLR II) with different levels of the coefficient of relative risk aversion. We estimate the following process for the consumption dynamics: $\Delta c_{t+1} = \mu + x_t + \sigma_\eta \eta_{t+1}$, $x_{t+1} = \rho x_t + \sigma_e e_{t+1}$. $\Delta x = \log(X_t) - \log(X_{t-1})$, and $\sigma[X]$ denotes the standard deviation of variable X . We use annual U.S. data from 1929 to 1998 from the Bureau of Economic Analysis. The sample is the same as in Bansal and Yaron (2004). Under Panel B we report the calibration of the exogenous consumption process Bansal and Yaron use. All values reported in the table are quarterly.

Statistic	Transitory Shocks		Permanent Shocks		
	$\varphi = 0.95$		$\varphi = 1.00$		
	$\psi = 0.05$	$\psi = 0.05$	$\psi = 1.5$	$\psi = 1.5$	
	$\beta = 1.064$	$\beta = 1.064$	$\beta = 0.998$	$\beta = 0.998$	
	$\xi = 0.70$	$\xi = 0.70$	$\xi = 18.0$	$\xi = 18.0$	
	$\sigma_z = 2.59\%$	$\sigma_z = 2.59\%$	$\sigma_z = 2.63\%$	$\sigma_z = 2.63\%$	
	$\gamma = \mathbf{5}$	$\gamma = \mathbf{25}$	$\gamma = \mathbf{5}$	$\gamma = \mathbf{25}$	
Panel A: Macroeconomic Moments (Quarterly)					
	U.S. Data 1929-1998				
$\sigma[\Delta y]$ (%)	2.62	2.61	2.61	2.64	2.64
$\sigma[\Delta c]/\sigma[\Delta y]$	0.52	0.52	0.45	0.52	0.43
$\sigma[\Delta i]/\sigma[\Delta y]$	3.32	2.36	2.12	1.83	1.60
Panel B: Consumption Dynamics: $\Delta c_{t+1} = \mu + x_t + \sigma_\eta \eta_{t+1}$, $x_{t+1} = \rho x_t + \sigma_e e_{t+1}$.					
	Bansal, Yaron Calibration				
$\sigma[\Delta c]$ (%)	1.360	1.360	1.129	1.362	1.123
$\sigma[x]$ (%)	0.172	0.126	0.120	0.336	0.342
ρ	0.938	0.959	0.959	0.970	0.974

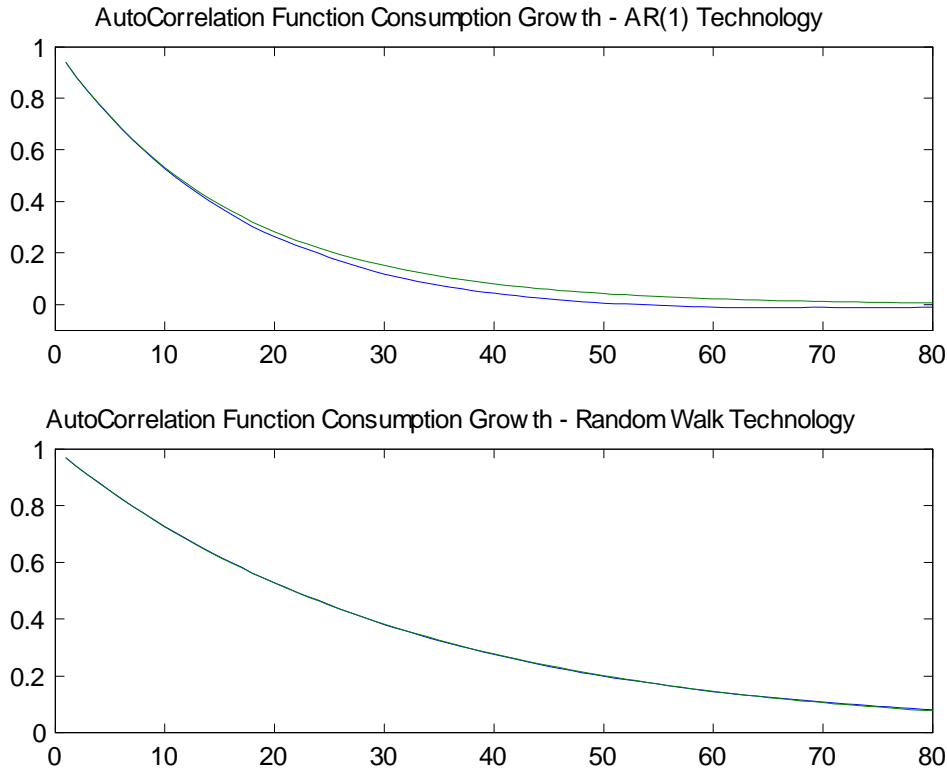


Figure 7: **Autocorrelation Functions Consumption Growth** Comparison of the autocorrelation function obtained directly from simulated data of models LLR I and LLR II to the autocorrelation function implied by the postulated process for expected consumption growth which we have estimated from the same simulated data from the respective models.

identical to the one assumed by Bansal and Yaron (2004) as driving process of their exchange economy model. Our results therefore provide a theoretical justification for their particular exogenous consumption growth process assumption. To evaluate whether the above specified process is a good approximation of the true consumption growth dynamics we first estimate the process from simulated data for a whole range of different model calibrations both with random walk- as well as with AR(1) technology processes. Then we compare the autocorrelation function obtained directly from the simulated data to the one implied by the above specified process which we have imposed on the data.

For the random walk technology the autocorrelation functions are virtually indistinguishable in all cases we have examined. Figure 7 shows this for the LLR II Model (permanent

technology shocks).²³ For the AR(1) technology the approximation turns out to get worse the lower the persistence of the driving process. Figure 7 shows the autocorrelation functions for Model LLR I (transitory technology shocks). A look at Figure 1 makes clear why the above specified approximation for the dynamics of the endogenous process for consumption is worse for the case where technology shocks are transitory, because the impulse-response of consumption to technology shocks is sometimes "hump-shaped". We therefore conclude that our postulated process is a good representation of the endogenous consumption growth dynamics for models with highly persistent technology shocks.²⁴

8.4 Numerical Solution

8.4.1 Solution Algorithm

We solve the following model:

$$V(K_t, Z_t) = \max_{C_t, K_{t+1}} \left\{ \left[(1 - \beta) C_t^{\frac{1-\gamma}{\theta}} + \beta (E_t [V(K_{t+1}, Z_{t+1})^{1-\gamma}])^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \right\}, \quad (53)$$

$$K_{t+1} = (1 - \delta) K_t + \phi \left(\frac{I_t}{K_t} \right) K_t, \quad (54)$$

$$I_t = Y_t - C_t, \quad (55)$$

$$Y_t = Z_t^{(1-\alpha)} K_t^\alpha, \quad (56)$$

$$\ln Z_{t+1} = \varphi \ln Z_t + \varepsilon_{t+1}, \quad (57)$$

$$\varepsilon_t \sim N(\mu, \sigma_z). \quad (58)$$

We focus in this appendix on the case where $\varphi = 1$. Since then the process for productivity is non-stationary, we need to normalize the economy by Z_t , in order to be able to numerically solve the model. To be precise, we let $\widehat{K}_t = \frac{K_t}{Z_t}$, $\widehat{C}_t = \frac{C_t}{Z_t}$, $\widehat{I}_t = \frac{I_t}{Z_t}$, and substitute. In the

²³We assume the disturbance terms η and e to be i.i.d. normally distributed. The shocks we obtain when we estimate our postulated process for consumption growth from simulated data turn out to be very close to normal. They display mild heteroscedasticity.

²⁴This conclusion relies on the assumption that the consumption process is covariance-stationary, which it is since the production function is constant returns to scale and preferences are homothetic. The autocorrelation function is then one of the fundamental time series representations. See, e.g., Hamilton (1994).

so transformed model all variables are stationary. The only state variable of the normalized model is \widehat{K} .²⁵ We can work directly on the appropriately normalized set of equations and then re-normalize after having solved the model.²⁶

The value function is given by:

$$\widehat{V}(\widehat{K}_t) = \max_{\widehat{C}_t, \widehat{K}_{t+1}} \left\{ \left[(1 - \beta) \widehat{C}_t^{\frac{1-\gamma}{\theta}} + \beta \left(E_t \left[(e^{\varepsilon_{t+1}})^{1-\gamma} \left(\widehat{V}(\widehat{K}_{t+1}) \right)^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \right\}. \quad (59)$$

We parameterize the value function with a 5th order Chebyshev orthogonal polynomial over a 6×1 Chebyshev grid for the state variable \widehat{K} :

$$\Psi^A(\widehat{K}) = \widehat{V}(\widehat{K}). \quad (60)$$

We use the value function iteration algorithm. At each grid point for the state \widehat{K} , given a polynomial for the value function $\Psi_i^A(\widehat{K})$, we use a numerical optimizer to find the policy (\widehat{C}^*) that maximizes the value function:

$$\widehat{K}_{t+1}^* e^{\varepsilon_{t+1}} = \widehat{Y}_t - \widehat{C}_t^* + (1 - \delta) \widehat{K}_t, \quad (61)$$

$$\widehat{V}^*(\widehat{K}_t) = \left[(1 - \beta) \left(\widehat{C}_t^* \right)^{\frac{1-\gamma}{\theta}} + \beta \left(E_t \left[(e^{\varepsilon_{t+1}})^{1-\gamma} \left(\Psi_i^A(\widehat{K}_{t+1}^*) \right)^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (62)$$

where Gauss-Hermite quadrature with 5 nodes is used to approximate the expectations operator. We use a regression of \widehat{V}^* on \widehat{K} in order to update the coefficients of the polynomial for the value function and so obtain $\Psi_{i+1}^A(\widehat{K})$.

8.5 The Linear Factor Model

The log stochastic discount factor is:

$$m_{t+1} = \theta \ln \beta - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) r_{a,t+1}, \quad (63)$$

²⁵Note that Z is *not* a state variable of the normalized model. This is due to the fact that we assume the autoregressive coefficient of the process for productivity $\ln Z_{t+1} = \rho \ln Z_t + \varepsilon_{t+1}$ to be unity: $\rho = 1$. As a consequence, ΔZ is serially uncorrelated.

²⁶In the paper we also report results for models where $\rho < 1$. In this case we work directly on the above non-normalized set of equations. The state variables are then K and Z . The solution algorithm is identical to the case where $\rho = 1$.

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. The process for consumption growth is:

$$\Delta c_{t+1} = \mu + x_t + \sigma_\eta \eta_{t+1}, \quad (64)$$

$$x_{t+1} = \rho x_t + \sigma_e e_{t+1}, \quad (65)$$

$$\sigma_{\eta,e} = \text{corr}(\varepsilon_{t+1}, \eta_{t+1}). \quad (66)$$

For convenience, the shocks are normalized to have unit variance here, unlike in the main part of the paper. Linearizing the wealth-consumption ratio around its steady state, we obtain (see Campbell, 1999, for a detailed derivation):

$$r_{a,t+1} \approx \kappa_0 + \kappa_1 p c_{t+1} - p c_t + \Delta c_{t+1}, \quad (67)$$

where $p c_t$ is the log wealth-consumption ratio, $\kappa_1 = \frac{\exp(\overline{p c_t})}{1 + \exp(\overline{p c_t})} \approx 0.96$, and $\overline{p c_t}$ is the steady state log wealth-consumption ratio. Assuming log aggregate consumption growth Δc_{t+1} to follow (64), Bansal and Yaron (2004) show that the log wealth-consumption ratio can be written as:

$$p c_{t+1} \approx A_0 + A_1 x_{t+1}, \quad (68)$$

where

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}. \quad (69)$$

Since $0 < \rho < 1$, $0 < \kappa_1 \rho < 1$. Thus, $A_1 > (<) 0$ if $\psi > (<) 1$. We substitute for $r_{a,t+1}$ in the log stochastic discount factor (63):

$$\begin{aligned} m_{t+1} &\approx \theta \ln \beta - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) \kappa_0 \\ &\quad - (1 - \theta) \kappa_1 p c_{t+1} + (1 - \theta) p c_t - (1 - \theta) \Delta c_{t+1} \\ &\approx \theta \ln \beta - (1 - \theta) \kappa_0 - \left(1 - \theta + \frac{\theta}{\psi}\right) \Delta c_{t+1} \\ &\quad - (1 - \theta) [\kappa_1 A_0 + \kappa_1 A_1 x_{t+1} - A_0 - A_1 x_t] \\ &= \theta \ln \beta - (1 - \theta) \kappa_0 - \left(1 - \theta + \frac{\theta}{\psi}\right) \Delta c_{t+1} - (1 - \theta) A_0 \kappa_1 + (1 - \theta) A_0 \\ &\quad - (1 - \theta) [\kappa_1 A_1 (\rho x_t + \sigma_e e_{t+1}) - A_1 x_t] \\ &= \theta \ln \beta - (1 - \theta) \kappa_0 - (1 - \theta) A_0 \kappa_1 + (1 - \theta) A_0 - \left(1 - \theta + \frac{\theta}{\psi}\right) \Delta c_{t+1} \\ &\quad - (1 - \theta) A_1 \kappa_1 \rho x_t - (1 - \theta) A_1 \kappa_1 \sigma_e e_{t+1} + (1 - \theta) A_1 x_t. \end{aligned} \quad (70)$$

Let:

$$\alpha = \theta \ln \beta - (1 - \theta) \kappa_0 - (1 - \theta) A_0 \kappa_1 + (1 - \theta) A_0. \quad (71)$$

Then:

$$\begin{aligned} m_{t+1} &\approx \alpha - \gamma \Delta c_{t+1} - (1 - \theta) A_1 \kappa_1 \rho x_t - (1 - \theta) A_1 \kappa_1 \sigma_e e_{t+1} + (1 - \theta) A_1 x_t \\ &= \alpha - \gamma \Delta c_{t+1} + (1 - \theta) A_1 (1 - \kappa_1 \rho) x_t - (1 - \theta) A_1 \kappa_1 \sigma_e e_{t+1}. \end{aligned}$$

Write this as:

$$m_{t+1} \approx a - b_1 \Delta c_{t+1} - b_2 e_{t+1} - b_3 x_t, \quad (72)$$

where $b_1 = \gamma$, $b_2 = (1 - \theta) A_1 \kappa_1 \sigma_e > 0$, $b_3 = -(1 - \theta) A_1 (1 - \kappa_1 \rho) < 0$, since $(1 - \theta) A_1 = \frac{\gamma - \frac{1}{\psi}}{1 - \kappa_1 \rho}$. Thus, if $\gamma > \frac{1}{\psi}$, then $(1 - \theta) A_1 > 0$. By applying a standard log-linear first-order approximation (see, e.g., Yogo, 2006, for a similar application), the (not log) stochastic discount factor can be written as:

$$\frac{M_t}{E[M_t]} \approx 1 + m_t - E[m_t]. \quad (73)$$

This in turn implies a linear unconditional factor model (see Cochrane, 2001):

$$\begin{aligned} E[R_{i,t+1} - R_{0,t+1}] &= b_1 Cov(\Delta c_{t+1}, R_{i,t+1} - R_{0,t+1}) + b_2 Cov(e_{t+1}, R_{i,t+1} - R_{0,t+1}) \\ &\quad + b_3 Cov(x_t, R_{i,t+1} - R_{0,t+1}), \end{aligned} \quad (74)$$

where $R_{i,t}$ denotes the time t gross return on asset i , and $R_{0,t}$ denotes the time t gross return on a reference asset (the risk free rate). Recall that $b_1, b_2 > 0$, $b_3 < 0$. The sign of the price of risk of each factor depends on the covariance matrix of the factors. The permanent shock model predicts that $cov(\eta_{t+1}, e_{t+1}) > 0$, which we confirm in the data.