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A Search Theory of Rigid Prices

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Abstract

In this paper, I build a model marketplace populated by a finite number of sellers—each producing its own variety of the good—and a continuum of buyers—each searching for a variety he likes. Using the model, I study the response of a seller's price to privately observed fluctuations in its idiosyncratic production cost. I find that the qualitative properties of this response critically depend on the persistence of the production cost. In particular, if the cost is i.i.d., the seller's price does not respond at all. If the cost is somewhat persistent, the seller's price responds slowly and incompletely. If the cost is very persistent, the seller's price adjusts instantaneously and efficiently to all fluctuations in productivity. I argue that these findings can explain why the monthly frequency of a price change is so much lower for processed than for raw goods.

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1 Introduction

The frequency at which prices change varies dramatically across goods. For some goods such as potatoes, gasoline and airfares, the monthly frequency of a price change is higher

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than fifty percent. For other goods like restaurant meals, haircuts or concerts, the monthly frequency of a price adjustment is below ten percent. In front of such figures, it is natural to wonder how much of the difference in the frequency of price adjustment can be explained by differences in various characteristics of the goods. Using previously unpublished data from the BLS, Bils and Klenow (2004) find that the single most important explanatory variable is whether a good is raw or processed. Indeed, the monthly frequency of a price change is 34 percent higher if the good is raw than if it is processed.

What's so different about raw and processed goods? For one thing, in many markets for processed goods, each producer makes its own variety of the good and each buyer spends some time searching for one variety that he likes. On the contrary, in many of the markets for raw goods, once a buyer finds a variety he likes, he can purchase it from a number of different producers. In this paper, I ask whether this difference alone can qualitatively explain the difference in the frequency of price adjustment between these two classes of goods.

In order to answer this question, I build a model of the typical market for processed goods. Specifically, I consider a marketplace populated by a finite number of sellers—each producing its own variety of the good—and a continuum of buyers—each demanding at most one unit of the good per period. In this market, a buyer does not know whether he likes the variety produced by a certain seller unless he spends some time researching it. Using this model, I characterize the response of a seller's price to changes in the cost of producing its own variety and compare it with the equilibrium price dynamics that would emerge in a Walrasian market (the natural model for the raw goods market). The analysis is carried out under the assumption that—perhaps because of reputation concerns—a seller can credibly commit to a complete contingent price schedule which maps public histories into terms of trade.

As a preliminary step, I characterize the optimal price schedule when the seller's cost of production is public information. I find that the schedule is time inconsistent, i.e. after any history, it prescribes a price lower than the one that would maximize the seller's continuation profits. In addition, I find that the schedule is cost sensitive, i.e. after any history, it prescribes a higher price the higher is the contemporaneous realization of the cost of production. Intuitively, the schedule is time inconsistent because the seller obtains part of the benefit from charging a lower price at date t in advance (namely, through the increase in the number of buyers who search its variety in periods $1, 2, ...t-1$), but it bears the cost entirely at date t. Intuitively, the schedule is cost sensitive because the seller benefits more from investing in its customer base when the productivity is higher. Taken together, these two properties imply that the optimal price schedule would typically be non incentive-compatible if the seller had private information about its idiosyncratic cost of production. In particular, the seller would have the incentive to overreport the realization of its cost in order to extract some more rents from the customer base.

In the second part of the paper, I consider the more realistic case where the seller privately observe its idiosyncratic cost of production. I find that the qualitative properties of the optimal incentive-compatible schedule critically depend on the persistence of the seller's cost. In particular, if the cost is i.i.d. over time, the schedule prescribes rigid prices—i.e. the terms of trade do not change in response to fluctuations in the seller's productivity. If the cost is somewhat persistent, the schedule prescribes sticky prices—i.e. the terms of trade adjust slowly and incompletely (as compared to the symmetric information case) in response to changes in the seller's cost of production. Finally, if the cost is very persistent, the optimal price schedule is the same under symmetric and asymmetric information.

In order to develop some intuition about these findings, consider a seller that has realized a relatively low cost of production and entertains the idea of lying and announcing a higher cost. On the one hand, if it misreports its type, the seller can charge a higher price to its customers. Because of the time inconsistent nature of the pricing problem, this effect increases the seller's profits. On the other hand, if it misreports its type, the seller induces the market to form irrationally pessimistic expectations about future costs and prices. Obviously, this effect lowers the seller's profits. When the cost of production is i.i.d. over time, the second effect is mute and the seller correctly reports its type only if it can charge the same price independently from its productivity. When the cost is somewhat persistent, the second effect is active and the low-cost seller correctly reports its type as long as prices are not too responsive to productivity. Finally, when the cost is very persistent, the second effect is so strong that the optimal price schedule under symmetric information becomes incentive compatible.

In light of these results, I conclude that sellers making their own variety of the good and buyers having to search for one variety they like are defining characteristics of the processed goods market which—in some cases—are alone sufficient to explain why prices adjust less frequently than in the market for raw goods.

Related Literature. My paper contributes to the theoretical literature on price rigidity. In models of time-dependent pricing (Calvo 1983, Taylor 1980), it is assumed that—for whatever reason—a seller cannot change its price in every period. Obviously, this assumption implies that sometimes a seller's price does not adjust in response to a change in fundamentals. In models of state-dependent pricing (Caplin and Spulber 1987, Golosov and Lucas 2007), it is assumed that a seller has to pay a fixed menu cost in order to change its price. Obviously, this assumption implies that a seller's price doesn't adjust in response to sufficiently small changes in fundamentals. My theory of price rigidity differs from these in two respects. First, according to my theory, adjustment costs are not necessary to explain why prices remain constant in the face of *real* changes in fundamentals. Secondly, according to my theory, there is no reason why prices should not adjust to *nominal* changes in fundamentals. Given these fundamental differences, I see my paper more as a complement than a substitute for menu costs and staggered pricing.

A closer substitute to my paper is Athey, Bagwell and Sanchirico (2004), where another theory of real rigidities is advanced. Specifically, they show that—under certain conditions—an optimal collusion scheme requires sellers to keep their price constant in the face of privately observed fluctuations in their idiosyncratic cost of production. Whether their theory is more useful than mine for understanding the difference in the frequency of price adjustment across goods is an open question.

Secondly, my paper contributes to the literature on pricing in markets with search frictions. Diamond (1971) considers a product market where buyers have to search a seller in order to find out its price. In such a market, the price has only the role of *distributing* the gains form trade between the seller and the buyers who have matched with it. Therefore, no matter how small search frictions are, every seller charges the pure monopoly price. On the other hand, Montgomery (1991), Moen (1997) and Burdett Shi Wright (2001) consider a static product market where buyers can observe the sellers' prices before they decide where to search. Because of frictions, not every buyer who searches a seller gets served. In such a market, the price has the role of *allocating* the buyers' search effort across sellers. Therefore, in equilibrium, sellers charge a price that is typically lower than the monopoly level.

In this paper, I consider a dynamic product market in which buyers can observe the seller's current and future prices before they decide where to search. In such a market, the seller's price at date t allocates the buyers' search effort in periods $1, 2, ...t$ and distributes the gains from trade in period t. Therefore, as time passes, the price becomes progressively less allocative and more distributive and the seller would like to renege on its promise. In the context of the Burdett Mortensen (1998) model, Coles (2001) had already recognized this type of time inconsistency. Yet, I am the first to realize that time inconsistency creates an incentive problem when the seller has private information about its time-varying cost of production. Moreover, I am the first to characterize the qualitative properties of the solution to this incentive problem.

Finally, my paper relates to the literature on pricing in markets where customers face a fixed cost of switching from one seller to another (see Klemperer 1987, 1995 and Beggs Klemperer 1992). In fact, also in these markets, the seller's price has both an allocative and a distributive role. And also in these markets, the seller's problem is time inconsistent because the allocative role becomes progressively less important relatively to the distributive role. Yet, because this literature works with the assumption that sellers cannot precommit to future prices, it has never encountered the kind of incentive problem that is central to my paper.

Structure of the Paper. In Section 2, I describe the physical environment. In Section 3, I formulate the seller's problem when productivity shocks are perfectly observable and characterize the first-best price schedule. In Section 4, I begin by formulating the pricing problem when productivity shocks are privately observed by the seller. Then, I identify a condition on the persistence of productivity shocks which guarantees that the first-best schedule is incentive compatible under asymmetric information. Finally, I characterize the qualitative properties of the second-best price schedule when the incentive compatibility constraints are binding. Section 5 briefly concludes. All proofs are relegated in the Appendix.

2 The Environment

The market for an indivisible and perishable consumption good is populated by a finite number of sellers and a continuum of buyers with large measure. In period t each seller i can produce its variety of the good at the constant marginal cost $c_{i,t}$. This cost is an idiosyncratic random variable that can take either the relatively low value c_ℓ or the relatively high value c_h, $0 < c_\ell < c_h$. The probability of each realization depends on the seller's past productivity—namely, $\Pr(c_{i,t+1} = c_{i,t}) = \rho \geq \frac{1}{2}$. The seller maximizes the expected sum of profits discounted at rate $\beta \in (0,1)$. In period t, each buyer j can participate to the market by paying an opportunity cost of $z > 0$ utils. If the buyer decides to visit the market, finds a variety that he likes and purchases one unit of it at the price $p_{j,t}$, he receives the periodical utility $u - p_{j,t}$, $u \in (z + c_h, \infty)$. If the buyer visits the market and doesn't purchase the good, his periodical utility is normalized to zero. The buyer maximizes the expected sum of utilities discounted at rate β.

Buyers and sellers come together through a search and matching process. If buyer j searches seller i, the two parties match successfully (i.e. the buyer likes the seller's variety) with probability $\lambda_{i,t}$ and fail to match with probability $1 - \lambda_{i,t}$. In the first case, the buyer has the option to purchase one unit of the good from the seller in the current period and, as long as the match survives, in future periods. In the second case, the buyer does not trade in the current period and has to search for a seller in the next period. Because of congestion effects, I assume that the probability $\lambda_{i,t}$ is a decreasing function of the measure $q_{i,t}$ of buyers searching seller i in period t. Because of network effects, I assume that the probability $\lambda_{i,t}$ is an increasing function of the measure $n_{i,t}$ of buyers who purchased from seller i in period $t-1$. For the sake of analytical tractability, I assume that $\lambda_{i,t}$ only depends on the ratio between $q_{i,t}$ and $n_{i,t}$. In particular, the function λ maps $\mathbb{R}_t$ into $[0,1]$ and is such that $\lambda'(q/n) < 0$, $\lambda(0) = 1$ and $\lambda(\infty) = 0$. A match dissolves if the buyer is exogenously displaced from the market (an event that occurs with probability $\sigma \in (0,1)$ in each period), if he voluntarily decides to leave the seller to search elsewhere or if he stops actively trading with the seller.[1]

In period t, events unfold in four stages. In the first stage, each seller realizes its productivity shock and publishes its terms of trade. Moreover, existing matches are subject to the displacement shock. In the second stage, buyers observe the entire distribution of terms of trade. Based on this information, matched buyers decide whether to remain with their current provider, search elsewhere or leave the market altogether. Unmatched buyers decide whether to visit the market and, if so, which seller to search. In the third stage, new matches are formed. In the fourth and final stage, matched buyers demand the good and sellers produce it. Throughout the paper, I assume that sellers cannot price discriminate because buyers are anonymous.

3 Pricing with Publicly Observed Costs

The purpose of this paper is to formulate and solve the pricing problem of a seller that enters the market in period $t = 0$ with the cost of production c_0 and a base of customers of measure $n(c_0) > 0$. I assume that—perhaps because of reputational concerns—the seller can pre-commit to a sequence of state-contingent prices $\mathbf{p} = \{p(h^t)\}_{t=0}^{\infty}$, where h^t is the seller's public history up to date t. In this section, I also assume that the seller's idiosyncratic cost of production is publicly observed and therefore $h^t = c^t = \{c_0, c_1, ...c_t\}$.

3.1 Seller's Problem

Denote with $U(c^t)$ the expected lifetime utility for a buyer who is matched with the seller in period t, after the history c^t has been realized. In period t, the buyer trades with the

[1] This no recall assumption is typical in search theory because it greatly simplifies the dynamics of the buyers' problem (cf Burdett and Mortensen (1998), Burdett and Coles (2003), Fishman and Rob (1995)).

seller and receives the periodical utility $u - p(c^t)$. With probability $1-\sigma$, in period $t+1$ the buyer has the option of remaining matched to the seller and receiving the continuation utility $U(c^{t+1})$ or searching some other seller/market and receiving the continuation utility Z. With probability σ, in period $t+1$ the buyer is exogenously displaced from the market and he receives the continuation utility Z. Therefore, $U(c^t)$ is equal to

$$U(c^t) = u - p(c^t) + \beta \sum_{c^{t+1}} \Pr(c^{t+1}|c^t) \left[(1-\sigma) \max\left\{U(c^{t+1}), Z\right\} + \sigma Z\right]. \tag{1}$$

Notice that Z is greater than $(1-\beta)^{-1}z$ because the buyer is free to stay out of the market. Also, Z is smaller than $(1-\beta)^{-1}z$ because, at this value, the entry of new buyers in the market is infinitely elastic. Therefore, Z is equal to the present value of the flow cost of entry z.

Next, consider a buyer who decides to search the seller in period t, after the history c^t has been realized. With probability $\lambda(q(c^t)/n(c^t))$, the buyer matches successfully with the seller and receives the expected lifetime utility $U(c^t)$. With probability $1-\lambda(q(c^t)/n(c^t))$, the buyer does not match with the seller and receives the lifetime utility βZ. In expectation, the value of searching the seller in period t is smaller than Z—because buyers are free to enter the market and search any particular seller they like—and is greater than Z whenever $q(c^t) > 0$—because those $q(c^t)$ buyers are free to search elsewhere. Therefore, in equilibrium, the measure of buyers $q(c^t)$ searching the seller is such that

$$\lambda\left(q\left(c^t\right)/n\left(c^t\right)\right)\left[U\left(c^t\right) - \beta Z\right] + \beta Z \leq Z \tag{2}$$

and $q(c^t) \geq 0$ with complementary slackness condition. It is convenient to denote with $\theta(U(c^t))$ the ratio of buyers searching the seller $q(c^t)$ to old customers $n(c^t)$ that solves the equilibrium condition (2).

If the value $U(c^t)$ of being matched to the seller is smaller than the outside option Z, every single one of the $n(c^t)$ old customers leaves and no new customers arrive. If $U(c^t)$ is greater than Z, a fraction $1-\sigma$ of the seller's $n(c^t)$ old customers returns and $n(c^t)\cdot\theta(U(c^t))\cdot\lambda\left(\theta(U(c^t))\right)$ new customers arrive. Overall, the law of motion for the seller's customer base can be written as

$$n(\{c^t, c_{t+1}\}) = n\left(c^t\right) \cdot \left[1 - \sigma + \eta\left(U\left(c^t\right)\right)\right], \tag{3}$$

where $\eta: \mathbb{R} \to \mathbb{R}$ is a function that takes the value $\sigma - 1$ if $U < Z$ and $\theta(U)\cdot\lambda\left(\theta(U)\right)$ otherwise. I assume that $\lambda(\theta)$ is such that the function $\eta(U)$ is twice continuously differentiable, strictly increasing and weakly concave and that $\eta(\infty) \leq \beta^{-1} - (1-\sigma)$.[2]

[2] All these conditions on $\eta(U)$ are satisfied if, for example, the function $\lambda(\theta)$ is equal to $(1+\alpha\theta^\gamma)^{-1/\gamma}$, where γ lies between $[0,1]$ and α is strictly positive.

In period $t = 0$, the seller commits to the price schedule $\mathbf{p}$ that maximizes the expected discounted sum of profits taking as given the law of motion for the customer base, i.e.

$$\max_{\mathbf{p}} \sum_{t=0}^{\infty} \beta^t \left[\sum_{c^t} \Pr(c^t|c_0) n(c_t)[1-\sigma+\eta(U(c^t))][p(c^t)-c_t] \right], \text{ s.t.} \\ (1), (3) \text{ and } c_0,\ n(c_0) \text{ given.} \tag{SP1}$$

The sequence problem (SP1) has two remarkable properties. First, after any history c^t, the optimal schedule $\mathbf{p}$ maximizes the seller's continuation profits subject to providing the buyers' at least the lifetime utility $U(c^t)$. Secondly, after any history c^t, the price schedule that maximizes the seller's continuation profits subject to providing the buyers with $U(c^t)$ is independent from the customer base $n(c^t)$ and the maximized profits are proportional to $n(c^t)$. In the Appendix, I use these two properties to prove that the sequence problem (SP1) has an equivalent recursive-form representation. In the recursive problem, the state variables are the seller's cost of production c_i and the buyers' promised value U. The choice variables are the value V actually delivered to the buyers, $V \geq U$, the current price p and next period's promised values U'_j, $j = \{\ell, h\}$. The objective function is the sum of current profits $(1-\sigma+\eta(V)) \cdot (p - c_i)$ and discounted future profits $(1-\sigma+\eta(V)) \cdot \beta \cdot E[\Pi_j(U'_j)|c_i]$.

LEMMA 1: (Recursive Formulation) *Denote with* $\Pi_i(U)$ *the value function associated to the sequence problem (SP1) when* $c_0 = c_i$, $n(c_0) = 1$ *and* $U(c_0)$ *is constrained to be greater or equal than* U. *Then* $\Pi_i(U)$ *is the unique solution to the Bellman equation*

$$\Pi_i(U) = \max_{p,V,U'_j \geq Z} (1-\sigma+\eta(V)) \left[p - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i) \Pi_j(U'_j) \right], \text{ s.t.} \\ U \leq V = u - p + \beta \textstyle\sum_j \Pr(c_j|c_i) \left[(1-\sigma)U'_j + \sigma Z \right]. \tag{BE1}$$

Let $\{V_i(U), p_i(U), U'_{i|j}(U)\}$ *be the policy functions associated to the Bellman equation above. Then, for all histories* $c^t = \{c^{t-2}, c_i, c_j\}$, *the optimal price schedule is such that* $p(c^t)$ *is equal to* $p_j(\tilde{U}(c^t))$, *where* $\tilde{U}(c^t) = U'_{j|i}(\tilde{U}(c^{t-1}))$ *and* $\tilde{U}(c_0) = Z$.

3.2 First-Best Price Schedule

After substituting out the price p, the recursive problem (BE1) can be broken down in two stages, i.e.

$$\Pi_i(U) = \max_V (1-\sigma+\eta(V)) \cdot \pi_i(V), \\ \pi_i(V) = u - c_i - V + \beta\sigma Z + \max_{U'_j \geq Z} \textstyle\sum_j \Pr(c_j|c_i) \left[\Pi_j(U'_j) + (1-\sigma)U'_j \right]. \tag{4}$$

In the first stage, the seller decides how much lifetime utility V its customers should be offered subject to the *promise-keeping* constraint $V \geq U$. In the second stage, the seller decides how the lifetime utility V should be allocated over time and across states.

How much lifetime utility should the seller offer to its customers? If V is smaller than the outside option Z, the seller does not have any customers and its profits are equal to zero. If V is greater than Z, the seller has $1-\sigma+\eta(V)$ customers and obtains the profit $\pi_i(V)$ from each one of them. Over this region, the seller's total profits $(1-\sigma+\eta(V))\cdot\ \pi_i(V)$ are first positive and increasing and then decreasing in the lifetime utility V. They are maximized at $\underline{U}_i$, where the benefit of attracting $\eta'(\underline{U}_i)$ additional new customers is equal to the cost of lowering the current price by 1 dollar, i.e.

$$\eta'(\underline{U}_i)\cdot\pi_i(\underline{U}_i) = 1-\sigma+\eta(\underline{U}_i) \tag{5}$$

The seller's offer is subject to the promise-keeping constraint $V \geq U$. If U is lower than $\underline{U}_i$, the constraint is moot and the seller offers the profit-maximizing value $\underline{U}_i$. If U is greater than $\underline{U}_i$, the constraint binds and the seller offers its customers the value it had promised them.

How should the seller allocate the buyers' lifetime utility V over time and across states? The seller can backload any feasible allocation by reducing the utility $u-p$ offered to its customers in the current period by $\Pr(c_j|c_i)\beta(1-\sigma)$ dollars and increasing their continuation value U'_j by 1 dollar. Then, in the current period, the seller collects $\Pr(c_j|c_i)\beta(1-\sigma)$ extra dollars per unit of output sold. And, in the next period, it attracts $\eta'(U'_j)$ additional customers and lowers the price by 1 dollar. If the seller frontloads a feasible allocation, the effects on current and future profits have the same magnitude and the opposite sign. The optimal allocation $(u-p, U'_\ell, U'_h)$ is such that the seller's profits cannot be increased by tilting the timing of benefits neither back nor forth, i.e.

$$-(1-\sigma) = \eta'(U'_j)\cdot\pi_j(U'_j) - (1-\sigma+\eta(U'_j)),\ \text{for } j=\ell,h, \tag{6}$$

$$p(V) = u - V + \beta\textstyle\sum_j \Pr(c_j|c_i)\left[(1-\sigma)U'_j + \sigma Z\right]. \tag{7}$$

Notice that, because an increase in U'_j by 1 util allows the seller to not only attract $\eta'(U'_j)$ additional customers in the next period but also raise its current price, the optimal continuation value U'_j is greater than $\underline{U}_j$.

Using the solution to the first and second stage problems, I can recover the structure of the first-best price schedule $\boldsymbol{p}$ and its qualitative properties. In period $t=0$, the seller enters the market with no prior obligations, $U(c^0)=Z$, and the production cost $c_0=c_i$. The seller offers its customers the profit-maximizing lifetime utility $\underline{U}_i$ by setting the current period's price to $p(\underline{U}_i)$ and committing to the continuation values (U'_ℓ, U'_h). In period $t \geq 1$, after the history c^t has been realized, the seller has the production cost $c_t = c_j$ and an obligation to deliver its customers at least U'_j. The seller offers them the promised lifetime utility U'_j by setting the current period's price to $p(U'_j)$ and committing to the continuation values (U'_ℓ, U'_h). Because U'_i is greater than $\underline{U}_i$, prices are decreasing over time. Also, because $\underline{U}_\ell$ is greater than $\underline{U}_h$ and U'_ℓ is greater than U'_h, prices are increasing in the contemporaneous realization of the cost of production.

Proposition 1: (Pricing with Publicly Observed Costs). *When the seller's idiosyncratic cost of production is publicly observed, the optimal price schedule $p = \{p\left(c^t\right)\}_{t=0}^{\infty}$ prescribes the price $p(c_0) = p(\underline{U}_i)$ for $c_0 = c_i$ and the price $p(c^t) = p(U'_i)$ for $t \geq 1$ and $c^t = \{c^{t-1}, c_i\}$. Keeping the cost of production constant, the prescribed prices are decreasing over time: $p(c^t) < p(c_0)$ for $t \geq 1$, $c^t = \{c^{t-1}, c_i\}$ and $c_0 = c_i$. Keeping the calendar date constant, the prescribed prices are increasing in the seller's production cost: $p(c_1^t) > p(c_2^t)$ for $c_1^t = \{c_1^{t-1}, c_h\}$ and $c_2^t = \{c_2^{t-1}, c_\ell\}$.*

The first-best price schedule characterized in Proposition 1 is *time-inconsistent*. At date $t = 0$, the seller finds optimal to charge its customers the high price $p(\underline{U}_i)$ and promise them the low price $p(U'_i)$ for the subsequent period. When date $t = 1$ arrives, the seller has already obtained part of the benefit of promising $p(U'_i)$—i.e. the increase in the inflow of new customers at $t = 0$—but has still to bear its entire cost. Then, the seller would like to renege the original schedule and, once again, charge its customers the high price $p(\underline{U}_i)$ and promise them the low price $p(U'_i)$ in the future.

4 Pricing with Privately Observed Costs

Consider a seller that enters the market in period $t = 0$ with the cost of production c_0 and a customer base of measure $n(c_0) > 0$. Assume that the seller can commit to a sequence of state-contingent prices $\boldsymbol{p} = \{p(h^t)\}_{t=0}^{\infty}$, where h^t is the seller's public history up to date t. Assume that, in every period $t \geq 1$, the seller privately observes the realization of its cost of production c_t and makes a public announcement $\widehat{c}_t \in \{c_\ell, c_h\}$ about it. Hence, h^t is $\widehat{c}^t = \{c_0, \widehat{c}_1, ...\widehat{c}_t\}$. In this section, I formulate and solve the pricing problem of the seller subject to the restriction that, after any history $\widehat{c}_t$, the customer's beliefs about the cost of production c_t are degenerate.

4.1 Seller's Problem

Without loss in generality, I can assume that the buyers interpret the seller's reports as truthful, i.e. $\Pr(c_t = \hat{c}_t | \widehat{c}^{t-1}) = 1$. Denote with $U(\widehat{c}^t)$ the expected lifetime utility for a buyer who is matched with the seller in period t, after the history of announcements $\widehat{c}^t = \{\widehat{c}^{t-1}, c_j\}$ has been reported. In period t, the buyer trades with the seller and receives the periodical utility $u - p(\widehat{c}^t)$. In period $t+1$,the buyer expects that the seller will report the production cost c_j and offer him the continuation utility $U(\{\widehat{c}^t, c_j\})$ with probability $\rho \geq 1/2$. The buyer expects that the seller will report the production cost c_{-j} and offer him the continuation utility $U(\{\widehat{c}^t, c_{-j}\})$ with probability $1-\rho$. Given those beliefs, $U(\widehat{c}^t)$ is equal to

$$U(\widehat{c}^t) = u - p(\widehat{c}^t) + \beta \left[\sum_{\widehat{c}^{t+1}} \Pr(\widehat{c}^{t+1} | \widehat{c}^t)[(1-\sigma) \max\{U(\widehat{c}^{t+1}), Z\} + \sigma Z] \right]. \tag{8}$$

Along the equilibrium path, the seller's reporting strategy must be consistent with the buyers' inference of the production cost c_t from the announcement $\hat{c}_t$. Therefore, for all $\widehat{c}^{t-1} = c^{t-1}$ and $c_t = c_i$, the price schedule $\boldsymbol{p}$ must induce the seller to report its type correctly, i.e.

$$\begin{gathered} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \sum_{c^\tau} \Pr(c^\tau | c^t) n\left(c^\tau\right) \left[1 - \sigma + \eta(U\left(c^\tau\right))\right] \left[p\left(c^\tau\right) - c_\tau\right] \right\} \geq \\ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \sum_{c^\tau} \Pr\left(c^\tau | c^t\right) n\left(\widehat{c}\left(c^\tau\right)\right) \left[1 - \sigma + \eta(U\left(\widehat{c}\left(c^\tau\right)\right))\right] \left[p\left(\widehat{c}\left(c^\tau\right)\right) - c_\tau\right] \right\}, \end{gathered} \tag{IC}$$

where $\widehat{c}(c^\tau)$ is the public history $\{c^{t-1}, c_{-i}, c_{t+1}, ... c_\tau\}$. In writing the *incentive compatibility* constraint (IC), I have assumed that—independently from its period$-t$ announcement—the seller will find optimal to report its type correctly in any subsequent period $\tau \geq t$. This is the right assumption to make because the seller's expected profits from reporting its true type and from lying depend on the public history $\widehat{c}^{\tau-1}$ and on the cost of production c_τ but not on the previous realizations of productivity shocks $c^{\tau-1}$. Therefore, the same incentive compatibility constraint (IC) which guarantees that the seller will truthfully report c_τ after the history $\widehat{c}^{\tau-1}$ has been realized and reported, also guarantees that the seller will truthfully report c_τ after the history $\widehat{c}^{\tau-1}$ has been reported and some different history has been realized.

In general, the optimal incentive-compatible price schedule $\boldsymbol{p}^*$ need not be renegotiation proof, i.e. there may exist some histories after which the seller and its customers would agree to modifying $\boldsymbol{p}^*$. In order to rule out this possibility, I restrict attention to price schedules $\boldsymbol{p}$

such that, for any reported history $\hat{c}^t = c^t$ and for any feasible $\hat{\boldsymbol{p}}$[3], if $U(c^t|\hat{\boldsymbol{p}}) > U(c^t|\boldsymbol{p})$ then

$$\begin{aligned} &\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \sum_{c^\tau} \Pr\left(c^\tau|c^t\right) n(c^\tau|\boldsymbol{p})[1-\sigma+\eta\left(U\left(c^\tau|\boldsymbol{p}\right)\right)][p(c^\tau)-c_\tau] \right\} > \\ &\quad \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \sum_{c^\tau} \Pr\left(c^\tau|c^t\right) n(c^\tau|\hat{\boldsymbol{p}})[1-\sigma+\eta\left(U\left(c^\tau|\hat{\boldsymbol{p}}\right)\right)][\hat{p}(c^\tau)-c_\tau] \right\}. \end{aligned} \tag{RP}$$

Notice that—because the value of a price schedule depends on the reported history $\widehat{c}^{t-1}$ and on the production cost c_t, but does not depend on the realized history c^{t-1}—the *renegotiation proofness* constraint (RP) guarantees that $\boldsymbol{p}$ is ex-post efficient even if the seller has lied in some previous period $\tau \leq t-1$.

In period $t = 0$, the seller commits to the price schedule $\boldsymbol{p}$ that maximizes the expected discounted profits subject to the incentive compatibility and renegotiation proofness constraints, i.e.

$$\begin{aligned} &\max_{\boldsymbol{p}} \sum_{\tau=t}^{\infty} \beta^t \left\{ \sum_{c^t} \Pr(c^t|c_0) n(c_t)[1-\sigma+\eta(U(c^t))][p(c^t)-c_t] \right\}, \text{ s.t.} \\ &\quad \text{(IC), (RP) and } c_0,\ n(c_0) \text{ given.} \end{aligned} \tag{SP2}$$

The sequence problem (SP2) has two remarkable features. First, after any realized history c^t and reported history $\widehat{c}^t$, the optimal price schedule $\boldsymbol{p}$ satisfies (IC) and (RP) at all subsequent dates $\tau \geq t+1$. And, among all the feasible schedules, $\boldsymbol{p}$ is the one that maximizes the profits of a seller with production cost $\hat{c}_t$ subject to providing the buyers with a lifetime utility non-smaller than $U(\widehat{c}^t)$. Secondly, after any realized history c^t and reported history $\widehat{c}^t$, the feasible schedule that maximizes the seller's profits subject to providing the buyers with $U(\widehat{c}^t)$ is independent from the customer base $n(\hat{c}^t)$ and the maximized profits are proportional to $n(\hat{c}^t)$. Using these two properties, in the Appendix, I prove that the sequence problem (SP2) has an equivalent recursive-form representation.

Lemma 2: (Recursive Formulation) *Denote with* $\Pi_i(U)$ *the value function associated to the sequence problem (SP2) when* $c_0 = c_i$, $n(c_0) = 1$ *and* $U(c_0)$ *is constrained to be greater or equal than* U. *Then* $\Pi_i(U)$ *solves the Bellman equation*

$$\begin{aligned} &\Pi_i(U) = \max_{p,V,U'_j \geq Z} (1-\sigma+\eta(V)) \left[p - c_i + \beta \sum_j \Pr(c_j|c_i) \Pi_j(U'_j) \right], \text{ s.t.} \\ &\quad U \leq V = u - p + \beta \sum_j \Pr(c_j|c_i) \left[(1-\sigma)U'_j + \sigma Z \right], \\ &\quad \Pi_j(U'_j) \geq \tilde{\Pi}_{-j}(U'_{-j}) \text{ for } j = \ell, h, \\ &\quad \tilde{\Pi}_i(U) = (1-\sigma+\eta(V_i(U))) \left[p_i(U) - c_{-i} + \beta \sum_j \Pr(c_j|c_{-i}) \Pi_j(U'_{j|i}(U)) \right]. \end{aligned} \tag{BE2}$$

[3] The schedule $\hat{\boldsymbol{p}}$ is feasible if it satisfies the incentive-compatibility constraint (IC) and the renegotiation-proofness condition (RP) in all periods $\tau \geq t+1$.

Let $\{V_i(U), p_i(U), U'_{i|j}(U)\}$ be the policy functions associated to the solution $\Pi_i(U)$ of the Bellman equation above. Then, for all histories $c^t = \{c^{t-2}, c_i, c_j\}$, the optimal price schedule is such that $p(c^t)$ is equal to $p_j(\tilde{U}(c^t))$, where $\tilde{U}(c^t) = U'_{j|i}(\tilde{U}(c^{t-1}))$ and $\tilde{U}(c_0) = Z$.

4.2 Very Persistent Costs: Fully Flexible Prices

When it satisfies the incentive compatibility and renegotiation proofness constraints, the first-best schedule is the solution to the pricing problem under asymmetric information. Because the first-best schedule is ex-post efficient, the renegotiation proofness constraint (RP) is certainly satisfied. But because the schedule is time-inconsistent, the incentive compatibility constraint (IC) need not hold. In this subsection, I identify a necessary and sufficient condition on the persistence of productivity shocks which guarantees that the first-best schedule will be incentive compatible. For the sake of simplicity, I carry out the analysis under the assumption that η is approximately linear over the range of values promised by the seller.

Imagine that the seller realizes the high cost of production c_h after having announced the history $\hat{c}^{t-1}$. If it chooses to report the low cost c_ℓ instead of c_h, the seller lowers its price by $p(U'_h) - p(U'_\ell)$ dollars and attracts $\eta'(U'_\ell - U'_h)$ additional customers in period t. Because the first-best price schedule is history independent, the report $\hat{c}_t$ does not affect the dynamics of prices and customers in subsequent periods. Therefore, the seller reports its actual cost of production c_h if and only if

$$\begin{aligned} &\Pi_h(U'_h) - \tilde{\Pi}_\ell(U'_\ell) = \\ &\quad \{(1-\sigma+\eta(U'_\ell))\,[1-\beta(1-\sigma)\,(2\rho-1)] - \eta'\pi_h(U'_h)\}\,(U'_\ell - U'_h) = \\ &\quad \{(1-\sigma)\,[1-\beta\,(1-\sigma+\eta(U'_\ell))\,(2\rho-1)] + \eta'(U'_\ell - U'_h)\}\,(U'_\ell - U'_h) \geq 0 \end{aligned} \tag{9}$$

where the third line is obtained after substituting in the first order condition (6). Analytically, it is immediate to verify that the incentive compatibility constraint (9) is always satisfied. Intuitively, the seller has no incentive to report c_ℓ instead of c_h because this would imply lowering a price that, from its perspective in period t, is already too low.

Next, imagine that the seller realizes the low cost of production c_ℓ after having announced the history $\hat{c}^{t-1}$. If it chooses to report the high cost c_h instead of c_ℓ, the seller increases its price by $p(U'_h) - p(U'_\ell)$ dollars and attracts $\eta'(U'_\ell - U'_h)$ fewer customers in period t. The report $\hat{c}_t$ does not affect the dynamics of prices and customers in subsequent periods.

Therefore, the seller reports its actual cost of production c_ℓ if and only if

$$\begin{aligned}&\Pi_\ell(U'_\ell) - \tilde{\Pi}_h(U'_h) = \\ &\quad \{\eta'\pi_\ell(U'_\ell) - (1-\sigma+\eta(U'_h))\,[1-\beta\,(1-\sigma)\,(2\rho-1)]\}\,(U'_\ell - U'_h) = \\ &\quad \{\eta'(U'_\ell - U'_h) - (1-\sigma)\,[1-\beta\,(1-\sigma+\eta(U'_h))\,(2\rho-1)]\}\,(U'_\ell - U'_h) \geq 0.\end{aligned} \tag{10}$$

The incentive compatibility constraint (10) may be satisfied or violated depending on parameter values. In particular, there exists a critical level of persistence ρ^* of the cost of production such that the constraint (10) is satisfied if ρ is greater than ρ^* and is violated if ρ is below ρ^*. Intuitively, because the schedule is time inconsistent, the low cost seller would like to raise the current price. But by reporting c_h instead of c_ℓ, the seller not only increases the current price, it also makes customers irrationally pessimistic about the future terms of trade. And this side effect becomes stronger the more persistent costs of production are.

PROPOSITION 2: (Fully Flexible Prices) *There exists a* $\rho^* \in \left[\frac{1}{2}, 1\right]$ *such that, for all* $\rho > (<) \rho^*$, *the first-best price schedule is feasible and optimal* (not feasible) *when the seller has private information about its idiosyncratic cost of production.*

4.3 IID Costs: Rigid Prices

When the persistence of productivity shocks is lower than the critical level ρ^*, the first-best schedule violates the incentive compatibility constraint (IC). In order to characterize the second-best schedule, it is convenient to break down the recursive problem (BE2) in two stages

$$\begin{aligned}&\Pi_i(U) = \max_{V\geq U}(1-\sigma+\eta(V))\cdot\pi_i(V), \\ &\pi_i(V) = u - c_i - V + \beta\sigma Z + \max_{U'_j\geq Z} \quad \textstyle\sum_j \Pr(c_j|c_i)\left[\Pi_j(U'_j) + (1-\sigma)U'_j\right], \text{ s.t.} \\ &\qquad\qquad \Pi_j(U'_j) \geq \tilde{\Pi}_j(U'_j) \text{ for } j\in\{\ell,h\}.\end{aligned} \tag{11}$$

In the first-stage problem, the choice variable is the customers' lifetime utility V. The objective function is the expected discounted profit for a seller with the current cost of production c_i. The function is first increasing and then decreasing in V and attains its unique maximum at $\underline{U}_i$, where $\underline{U}_i$ is the solution to the equation (5). The choice of V is limited by the promised-keeping constraint $U \leq V$. Therefore, if $U \leq \underline{U}_i$, the solution to the first-stage problem is to provide customers with the profit-maximizing value $\underline{U}_i$. If $U > \underline{U}_i$, the solution is to provide customers with the promised value U.

In the second-stage problem, the choice variables are the customers' continuation values U'_ℓ and U'_h. The objective function is the profit per customer for a seller that provides them with the lifetime utility V. The function is quasi-concave in (U'_ℓ, U'_h) and attains its unique maximum at (U'^*_ℓ, U'^*_h), where U'^*_j is the solution to the equation (6). The choice of (U'_ℓ, U'_h) is limited by the incentive-compatibility constraint $\Pi_j(U'_j) \geq \tilde{\Pi}_{-j}(U'_{-j})$. Because V enters the objective function separately from the choice variables, the solution to the second-stage problem is independent from the lifetime utility V and can be denoted with $(U'_{\ell|i}, U'_{h|i})$. Because the objective function is increasing in U'_j and the constraint is independent from U'_j for all $U'_j \leq \underline{U}_j$, the solution to the second-stage problem $(U'_{\ell|i}, U'_{h|i})$ is greater than $(\underline{U}_\ell, \underline{U}_h)$.

Using the qualitative properties of the solution to the first and second stage problems, I can express the incentive compatibility constraint as

$$\begin{aligned} &(1-\sigma+\eta(U'_j)) \cdot \pi_j(U'_j) \geq (1-\sigma+\eta(U'_{-j})) \cdot \tilde{\pi}_{-j}(U'_{-j}), \\ &\tilde{\pi}_i(U) \equiv u - c_{-i} - U + \beta \textstyle\sum_j \left\{ \Pr(c_j|c_i) \left[(1-\sigma)U'_{j|i} + \sigma Z \right] + \Pr(c_j|c_{-i}) \Pi_j(U'_{j|i}) \right\}. \end{aligned} \tag{12}$$

Notice that, if the seller realizes the production cost c_{-i} but announces c_i, its expected profits per customer $\tilde{\pi}_i(U)$ are *generally* different from $\pi_{-i}(U)$. First, when the seller misreports its type, the customers' expectations about the value of the match are not correct. In particular, while the customers expect to receive the continuation value $U'_{i|i}$ with probability ρ and $U'_{-i|i}$ with probability $1-\rho$, the seller offers them $U'_{i|i}$ a fraction $1-\rho$ of the time and $U'_{-i|i}$ a fraction ρ of the time. Secondly, when the seller misreports its type, the continuation values $U'_{\ell|i}$ and $U'_{h|i}$ prescribed by the second-best schedule are not its preferred way to allocate the customers' lifetime utility over time and across states.

Only when costs are i.i.d., $\tilde{\pi}_i(U)$ is equal to $\pi_{-i}(U)$. In this case, a seller that realizes the cost of production c_j correctly reports its type if it attains higher profits by offering to its customers the lifetime utility U'_j rather than U'_{-j}, i.e.

$$(1-\sigma+\eta(U'_j)) \cdot \pi_j(U'_j) \geq (1-\sigma+\eta(U'_{-j})) \cdot \pi_j(U'_{-j}). \tag{13}$$

Since U'_h and U'_ℓ are both greater than $\underline{U}_h$, a seller that realizes the cost of production c_h announces its true type if and only if U'_h is smaller than U'_ℓ. Since U'_ℓ is greater than $\underline{U}_\ell$ but U'_h need not be greater than $\underline{U}_\ell$, a seller that realizes the cost of production c_ℓ announces its true type if either U'_h is greater than U'_ℓ or sufficiently smaller than the profit-maximizing value $\underline{U}_\ell$. The set of continuation values (U'_ℓ, U'_h) that induces both seller's types to report their actual costs is illustrated in Figure 1.

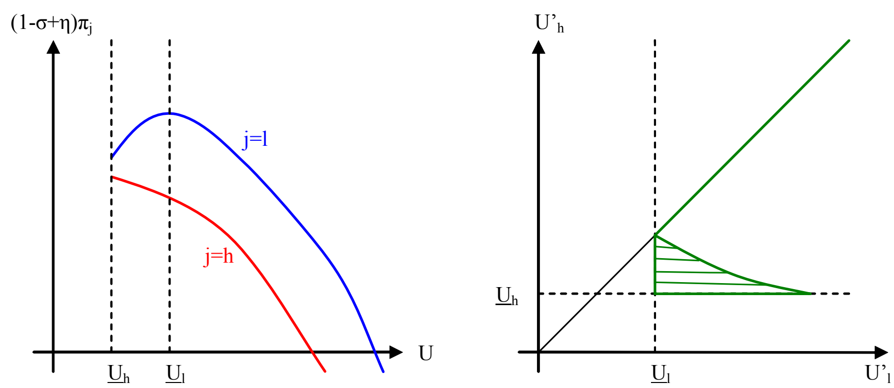

Figure 1: Incentive compatible continuation values (IID Shocks)

Given the characterization of the incentive compatibility constraint (13), I can conclude that there exist two candidate solutions to the second-stage problem. The first solution prescribes that the continuation value offered to the customers should be independent from the seller's announcement about its cost of production, i.e. $U'_h = U'_\ell = U'$, and such that the allocation of customers' utility over time is on average efficient

$$-(1-\sigma) = \sum_j \tfrac{1}{2}\left[\eta'(U') \cdot \pi_j\left(U'\right) - 1 - \sigma + \eta(U')\right]. \tag{14}$$

The second solution prescribes that the continuation value should be lower when the cost of production announced by the seller is higher, i.e. $U'_h < U'_\ell$, and that U'_h should be sufficiently far below $\underline{U}_\ell$ to induce the low-cost seller to truthfully report its type. When productivity shocks are small, the state-independent solution is optimal because it closely approximates the first-best (U'^*_ℓ, U'^*_h) while the state-contingent solution approximates the no commitment outcome $(\underline{U}_\ell, \underline{U}_h)$. This leads to the following proposition.

Proposition 3 (Rigid Prices) *Let $c_\ell = c - \Delta$ and $c_h = c + \Delta$ for some $c \in (0, u - z)$. When the seller's cost of production is privately observed and i.i.d. over time, there exists a $\Delta^* > 0$ such that, for all $\Delta \in (0, \Delta^*)$, the optimal price schedule $\{p(\hat{c}^t)\}_{t=0}^{\infty}$ prescribes the price $p(\hat{c}^t) = u - U' + \beta[(1-\sigma)U' + \sigma Z]$ for all dates $t \geq 1$ and all histories $\hat{c}^t$.*

4.4 Moderately Persistent Costs: Sticky Prices

In this subsection, I characterize the optimal price schedule under asymmetric information when production costs are positively correlated over time, but not to the point where the first-best schedule becomes feasible. In order to develop the analysis, I find convenient to first solve a version of the second-stage problem (11) that abstracts from the incentive compatibility constraint for the high-cost seller and to later verify that the constraint is satisfied.

Let the persistence ρ of production costs be anywhere in the interval $(0, \rho^*)$. The relaxed version of the second-stage problem in (11) is

$$\begin{aligned} \pi_i(V) = \quad & u - c_i - V + \beta\sigma Z + \max_{U'_j \geq \underline{U}_j} \sum_j \Pr(c_j|c_i) \left[\Pi_j(U'_j) + (1-\sigma)U'_j\right], \text{ s.t.} \\ & (1 - \sigma + \eta(U'_\ell)) \cdot \pi_\ell(U'_\ell) \geq (1 - \sigma + \eta(U'_h)) \cdot \widetilde{\pi}_h(U'_h). \end{aligned} \tag{15}$$

Consider the incentive compatibility constraint for the low-cost seller. On the one hand, the seller's profits from truthfully reporting its type are monotonically decreasing with the continuation value U'_ℓ promised to its customers. On the other hand, the seller's profits from misreporting its type are first increasing and then decreasing in the continuation value U'_h expected by the customers and they are maximized at $\underline{\tilde{U}}_h \in [Z, \underline{U}_\ell]$. Moreover, when $U'_\ell = U'_h$, the seller makes higher profits by correctly reporting its type rather than lying. Therefore, the incentive compatibility constraint is satisfied either when U'_h is not much smaller than the alternative continuation value U'_ℓ or when U'_h is sufficiently far below the profit-maximizing value $\underline{\tilde{U}}_h$. The set of incentive-compatible continuation values is illustrated in Figure 2.

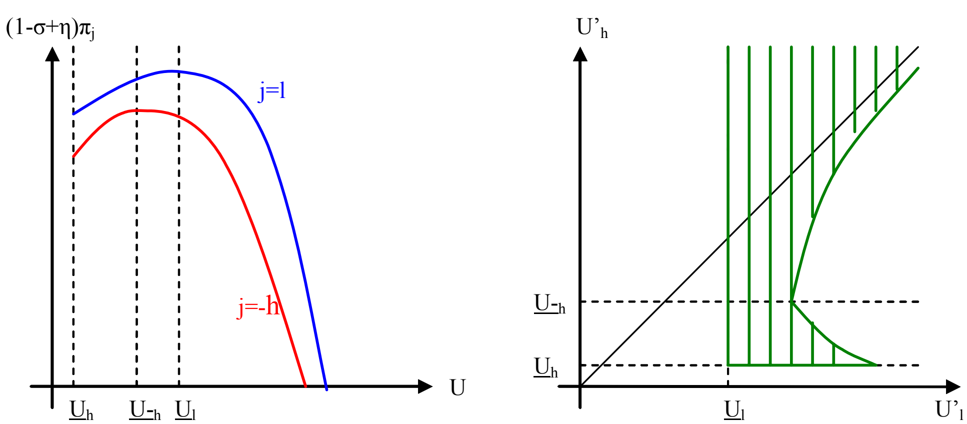

Figure 2: Incentive compatible continuation values (Persistent Shocks)

When $\rho < \rho^*$, the first-best solution (U'^*_ℓ, U'^*_h) to the second-stage problem does not satisfy the low-cost seller's incentive compatibility constraint. The second-best solution distorts the continuation values (U'_ℓ, U'_h) away from (U'^*_ℓ, U'^*_h) in order to make the low-cost seller indifferent between correctly reporting its type and lying. More specifically, when productivity shocks are small, the second-best solution distorts the continuation value U'_ℓ downward and U'_h upward. And, if in the previous period the seller has reported the low cost of production, then U'_ℓ is distorted less and U'_h is distorted more than if the seller had reported c_h. Overall, the second-best continuation values $(U'_{\ell|i}, U'_{h|i})$ are such that $U'^*_h \leq U'_{h|i} < U'_{\ell|i} \leq U'^*_{\ell|i}$, $U'_{\ell|\ell} > U'_{\ell|h}$ and $U'_{h|h} < U'_{h|\ell}$.

Now, I am in the position to recover the structure of the second-best price schedule $\boldsymbol{p} = \{p(\widehat{c}^t)\}_{t=0}^{\infty}$. In period $t = 0$, the seller enters the market with no prior obligations, $U(c_0) = Z$, and the production cost c_i. The seller offers its customers the profit-maximizing lifetime utility $\underline{U}_i$ by setting the current period's price to $p_i(\underline{U}_i)$ and committing to the continuation values $(U'_{\ell|i}, U'_{h|i})$, where

$$p_i(U) = u - U + \beta \textstyle\sum_j \Pr(c_j|c_i)\,[(1-\sigma)\,U'_{j|i} + \sigma Z]. \tag{16}$$

In period $t \geq 1$ and after the public history $\hat{c}^{t-1} = \{\hat{c}^{t-2}, c_i\}$ has been realized, the seller reports its actual production cost c_j and offers its customers the promised lifetime utility $U'_{j|i}$ by setting the current period's price to $p_j(U'_{j|i})$ and committing to the continuation values $(U'_{\ell|j}, U'_{h|j})$.

From the properties of the continuation values $U'_{j|i}$, I can characterize the joint dynamics of costs and prices. First, "steady-state" prices are increasing in the production cost. That is, if the seller realizes the production cost c_ℓ for a sufficiently long period of time, it charges the price $p_\ell(U'_{\ell|\ell})$ which is strictly lower than the price $p_h(U'_{h|h})$ it would have charged if it had realized c_h instead. Secondly, prices are "sticky." That is, when the seller first realizes the high production cost, it charges a price $p_h(U'_{h|\ell})$ which is strictly lower than the steady-state level $p_h(U'_{h|h})$. Conversely, when the seller first realizes the low production cost, it charges a price $p_\ell(U'_{\ell|h})$ which is strictly greater than $p_\ell(U'_{\ell|\ell})$.

Proposition 4: (Sticky Prices) *Let $c_\ell = c - \Delta$ and $c_h = c + \Delta$ for some $c \in (0, u - z)$ and suppose that the solution to the Bellman equation (BE2) is unique. Then, when the seller's cost of production is privately observed and persistent ($\rho > 1/2$), there exists a $\Delta^* > 0$ such that, for all $\Delta \in (0, \Delta^*)$, the optimal price schedule $\{p(\hat{c}^t)\}_{t=0}^{\infty}$ prescribes the price $p(c_0) = p_i(\underline{U}_i)$ for $c_0 = c_i$ and the price $p(\hat{c}^t) = p_j(U'_{i|j})$ for $t \geq 1$ and $c^t = \{c^{t-2}, c_i, c_j\}$. The prescribed price is increasing in c_i if the seller has reported c_i for two or more periods: $p(c^t_1) > p(c^t_2)$ for $c^t_1 = \{c^{t-2}_1, c_h, c_h\}$ and $c^t_2 = \{c^{t-2}_2, c_\ell, c_\ell\}$. The prescribed price adjusts* *slowly over time in response to a change in the reported cost of production:* $p(c^{t+1}) > p(c^t)$ *for* $c^{t+1} = \{c^{t-2}, c_\ell, c_h, c_h\}$ *and* $p(c^{t+1}) < p(c^t)$ *for* $c^{t+1} = \{c^{t-2}, c_h, c_\ell, c_\ell\}$.

5 Conclusions

In this paper, I have built a model marketplace populated by a finite number of sellers—each producing its own variety of the good—and a continuum of buyers—each searching for a variety he likes. Using the model, I have studied the response of a seller's price to privately observed fluctuations in its idiosyncratic production cost. I have found that the qualitative properties of this response critically depend on the persistence of the production cost. In particular, if the cost is i.i.d., the seller's price does not respond at all. If the cost is somewhat persistent, the seller's price responds slowly and incompletely. If the cost is very persistent, the seller's price adjusts instantaneously and efficiently to all fluctuations in productivity. I have argued that these findings can explain why the monthly frequency of a price change is so much lower for processed than for raw goods.

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A Appendix

A.1 Proof of Lemma 1

Claim 1: After any history c^t, the optimal price schedule $\mathbf{p}$ is such that $U(c^t|\mathbf{p}) \geq Z$.
Proof: On the way to a contradiction, let c_1^t be the earlier history at which $U(c_1^t|\mathbf{p}) \geq Z$. Then, if the history c_1^t is realized (an event which occurs with positive probability), the seller

loses all its current customers and can't attract any any new customers in the future. Its expected discounted profits are equal to zero. Now, consider an alternative schedule $\widehat{\mathbf{p}}$ such that $\widehat{p}(c^\tau) = p(c^\tau)$ if c^τ is not a subsequent of c_1^t and $\widehat{p}(c^\tau) = u - z > 0$ otherwise. For all $\tau < t$, the seller's periodical profits are the same with $\widehat{\mathbf{p}}$ and $\mathbf{p}$ because prices and customers are the same. Similarly, for all $c^t \neq c_1^t$, the seller's continuation profits are the same with $\widehat{\mathbf{p}}$ and $\mathbf{p}$. Finally for $c^t = c_1^t$, the continuation profits are strictly positive. Overall, in period $t = 0$, the seller strictly prefers to commit to the schedule $\widehat{\mathbf{p}}$ that $\mathbf{p}$, which contradicts the optimality of the latter. $\|$

Denote with $\Pi_i(U)$ the value function associated to the sequence problem (SP1) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c_0)$ is constrained to be *greater or equal* than U. Denote with $\Pi_i^+(U)$ the value function associated to (SP1) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c_0)$ is constrained to be *equal* to U.

Claim 2: The value functions $\Pi_i(U)$ and $\Pi_i^+(U)$ are such that

$$\begin{aligned} \Pi_i(U) &= \max_{V,p,U_j' \geq Z} (1-\sigma+\eta(V)) \left[p - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i) \Pi_j^+(U_j')\right], \text{ s.t.} \\ U &\leq V \equiv u - p + \beta \textstyle\sum_j \Pr(c_j|c_i) \left[(1-\sigma)U_j' + \sigma Z\right]. \end{aligned} \tag{A1}$$

Proof: Making use of Claim 1, I can write the value function $\Pi_i(U)$ as

$$\Pi_i(U) = \max_{\substack{p(c_0),\mathbf{p}_1,\\ U(c_0),U(c^1)}} n(c^1)\left[p(c^0) - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[\sum_{c^\tau} \Pr\left(c^t | \{c_0,c_j\}\right) \frac{n(c^{t+1})}{n(c^1)}[p(c^t) - c_t]\right]\right]$$

$$U \leq U(c^0) \equiv u - p(c^0) + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[(1-\sigma)U(\{c_0,c_j\}) + \sigma Z\right],$$

$$Z \leq U\left(c^t\right) \equiv u - p(c^t) + \beta \textstyle\sum_{c^{t+1}} \Pr(c^{t+1}|c^t)\left[(1-\sigma)U(c^{t+1}) + \sigma Z\right],$$

$$n(c^1) = (1-\sigma+\eta(U(c^0))),\ \frac{n(c^{t+1})}{n(c^1)} = \frac{n(c^t)}{n(c^1)}(1-\sigma+\eta(U(c^t))).$$

The maximization problem above can be broken down in two stages. In the first stage, the seller chooses $p(c^0), U(c^0)$ and $U(\{c_0,c_j\})$ subject to the first and third constraints. In the second stage, the seller chooses $\mathbf{p}_1 = \{p\left(c^t\right)\}_{t=1}^{\infty}$ in order to maximize its continuation profits subject to delivering *exactly* $U(\{c_0,c_j\})$ to the customers and given an initial cost of production c_j and a customer base with measure $n\left(c^1\right)/n\left(c^1\right) = 1$. Therefore, the value function associated to the second-stage problem is $\Pi_j^+(U(\{c_0,c_j\}))$. $\|$

Claim 3: The function $\Pi_i(U)$ satisfies the Bellman equation (BE1).
Proof: First, notice that the solution to the maximization problem in (A1) is a continuation value U_j^+ that belongs to the set $U_j^{PF} = \{U : \Pi_j^+(U) = \Pi_j(U)\}$, i.e. the set of continuation

values U such that the seller could not increase its profits by delivering more than U. Therefore, I can restrict attention to continuation values in U_j^{PF} and replace $\Pi_j^+(U)$ with $\Pi_j(U)$ in (A1). Secondly, notice that, if the continuation function in (A1) is $\Pi_j(U)$ and the choice of continuation values is not restricted to U_j^{PF}, the solution to the maximization problem is $U_j' \in U_j^{PF}$. Therefore, I can relax the choice set and obtain (BE1). ‖

Claim 4: If $P_i(U)$ is a solution to the functional equation $(BE1)$, then $P_i(U)$ is equal to $\Pi_i(U)$.
Proof: Denote with T the mapping associated to $(BE1)$. It is immediate to verify that T satisfies the Blackwell's sufficient conditions for a contraction mapping. Therefore, T has a unique fixed point. ‖

To conclude the proof of Lemma 1, notice that the value function associated to the sequence problem (SP1) when $c_0 = c_i$ and $n(c_0) > 0$ is given by $n(c_0) \cdot \Pi_i(0)$.

A.2 Proof of Proposition 1

Consider the first-stage problem in (4). For $V < Z$, the objective function $(1 - \sigma + \eta(V)) \cdot \pi_i(V)$ is equal to zero because $\eta(V) = \sigma - 1$. For $V = Z$, the function is strictly positive because $\eta(V) = 0$ and, as proved in Lemma 1, $\pi_i(Z) > 0$. For $V \geq Z$, the function is quasi-concave because it is concave wherever increasing and may be convex only when strictly decreasing. The function attains its maximum for $V = \underline{U}_i$, where $\underline{U}_i$ satisfies

$$[\eta'(\underline{U}_i) \cdot \pi_i(\underline{U}_i) - (1 - \sigma + \eta(\underline{U}_i))] \cdot (\underline{U}_i - Z) = 0. \tag{A2}$$

From the properties of the objective function, it follows that the solution $V_i(U)$ to the first-stage problem is $\underline{U}_i$ whenever $U \leq \underline{U}$ and U otherwise. In turn, the value function $\Pi_i(U)$ associated to the first-stage problem is constant at $(1 - \sigma + \eta(\underline{U}_i)) \cdot \pi(\underline{U}_i)$ whenever $U \leq \underline{U}_i$ and is strictly decreasing and quasi-concave otherwise.

Next, consider the second-stage problem in (4). As proved in Lemma 1, the choice set can be restricted to those continuation values that are greater than the profit-maximizing values $(\underline{U}_\ell, \underline{U}_h)$. Over this domain, the objective function is quasi-concave and attains is maximum at (U_ℓ', U_h'), where U_j' satisfies

$$\eta'(U_j') \cdot \pi_j(U_j') - (1 - \sigma + \eta(U_j')) = -(1 - \sigma). \tag{A3}$$

Because $\eta'\pi_j - (1-\sigma+\eta)$ is non-negative for all $U \leq \underline{U}_j$, the optimal continuation value U'_j is strictly greater than the profit-maximizing value $\underline{U}_j$.

Finally, I want to compare the solution to the first and second stage problem under high and low cost of production. Denote with T the contraction mapping associated to the Bellman equation (BE1). Since $(TP)_h < (TP)_\ell$ whenever $P_h \leq P_\ell$, the unique fixed point Π of the contraction mapping T associated to the Bellman equation (BE1) is such that the profit function is strictly decreasing in the cost of production. In turn, this implies that $\pi_h(U) < \pi_\ell(U)$ and, through the first order conditions (A2) and (A3), that $\underline{U}_h \leq \underline{U}_\ell$ and $U'_h < U'_\ell$.

A.3 Proof of Lemma 2

Claim 1: After any history $\widehat{c}^t$, the optimal price schedule $\boldsymbol{p}$ is such that $U\left(\widehat{c}^t|\boldsymbol{p}\right) \geq Z$.
Proof: Suppose that, at $\widehat{c}_1^t$, the optimal schedule $\boldsymbol{p}$ is such that the buyers' lifetime utility $U\left(\widehat{c}^t|\boldsymbol{p}\right)$ is strictly smaller than Z and, consequently, the seller's expected profits are zero. Consider the alternative schedule $\widehat{\mathbf{p}}$ which prescribes the constant price $\widehat{p}(\widehat{c}^\tau) = u - z$ for all histories $\widehat{c}^\tau$ that are subsequents of $\widehat{c}_1^t$. At $\widehat{c}_1^t$, the alternative schedule $\widehat{\boldsymbol{p}}$ is such that the buyers' lifetime utility $U\left(\widehat{c}^t|\widehat{\boldsymbol{p}}\right)$ is equal to Z and the seller's expected profits are strictly positive. The schedule $\widehat{\boldsymbol{p}}$ satisfies the incentive-compatibility constraint (IC). If it is also renegotiation-proof, then $\widehat{\boldsymbol{p}}$ is a feasible Pareto improvement over $\boldsymbol{p}$ after the history $\widehat{c}_1^t$ is realized. Therefore, $\boldsymbol{p}$ violates the constraint (RP) and is not an optimum. If $\widehat{\boldsymbol{p}}$ is not renegotiation-proof, then there exists a feasible schedule $\tilde{\boldsymbol{p}}$ which is a Pareto improvement over $\widehat{\boldsymbol{p}}$ and, a fortiori, over $\boldsymbol{p}$. Again, $\boldsymbol{p}$ violates the constraint (RP) and is not an optimum.
$\|$

Denote with $\Pi_i\left(U\right)$ the value function associated to (SP2) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c^0)$ is constrained to be *greater or equal* than U. Denote with $\Pi_i^+(U)$ the value function associated to (SP2) when $c_0 = c_i$, $n(c_0) = 1$ and $U(c^0)$ is constrained to be *equal* to U. Finally, let U^{PF} be the set of promised values U such that the seller could not increase its profits by delivering more than U, i.e. $U_i^{PF} = \{U : \Pi_i^+(U) = \Pi_i(U)\}$.

Claim 2: The value function $\Pi_i^+(U)$ satisfies the Bellman equation

$$\begin{aligned}
&\Pi_i^+(U) = \max_{p_i, U'_j \in U_j^{PF}} (1-\sigma+\eta(U))\left[p - c_i + \beta \textstyle\sum_j \Pr(c_j|c_i)\Pi_j^+(U'_j)\right], \text{ s.t.} \\
&U = u - p + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[(1-\sigma)U'_j + \sigma Z\right], \qquad \text{(A4)} \\
&\Pi_j^+(U'_j) \geq \tilde{\Pi}_{-j}^+(U'_{-j}),
\end{aligned}$$

where

$$\tilde{\Pi}_i^+(U) = (1-\sigma+\eta(U))\left[p_i^+(U) - c_{-i} + \beta \sum_j \Pr(c_j|c_{-i})\Pi_j^+(U'^+_{j|i}(U))\right].$$

Moreover, the value functions $\Pi_i(U)$ and $\Pi_i^+(U)$ satisfy the functional equation

$$\begin{aligned}
&\Pi_i(U) = \max_{V,p,U_j' \in U_j^{PF}} (1-\sigma+\eta(V))\left[p - c_i + \beta \sum_j \Pr(c_j|c_i)\Pi_j^+(U_j')\right], \text{ s.t.} \\
&U \leq V = u - p + \beta \sum_j \Pr(c_j|c_i)\left[(1-\sigma)U_j' + \sigma Z\right], \\
&\Pi_j^+(U_j') \geq \tilde{\Pi}_{-j}^+(U'_{-j}).
\end{aligned} \tag{A5}$$

Proof: The proof of this claim follows directly from the analysis of the seller's problem in Section 4.1. ‖

Claim 3: The function $\Pi_i(U)$ satisfies the Bellman equation (BE2).
Proof: The continuation value U_j' that solves the maximization problem in (A5) belongs to the set U_j^{PF}. For all $U \in U_j^{PF}$, the profit function $\Pi_j^+(U)$ is equal to $\Pi_j(U)$ and the function $\tilde{\Pi}_j^+(U)$ is equal to $\tilde{\Pi}_j(U)$, where $\tilde{\Pi}_j(U)$ is defined in (BE2). Therefore, I can replace the continuation profit $\Pi_j^+(U)$ with $\Pi_j(U)$ in the objective function of (A5) and substitute the incentive compatibility constraint $\Pi_j^+(U_j') \geq \tilde{\Pi}_{-j}^+(U'_{-j})$ with $\Pi_j(U_j') \geq \tilde{\Pi}_{-j}(U'_{-j})$. Moreover, if the constraint $U_j' \in U^{PF}$ is removed from the modified problem, the optimal continuation value U_j' belongs to U_j^{PF}. Therefore, I can also substitute the constraint $U_j' \in U_j^{PF}$ with the constraint $U_j \geq Z$. ‖

To conclude the proof of Lemma 2, notice that the value function associated to the sequence problem (SP2) when $c_0 = c_i$ and $n(c_0) > 0$ is given by $n(c_0) \cdot \Pi_i(0)$.

A.4 Proof of Proposition 2

For $\hat{c}^{t-1} = c^{t-1}$ and $c_t = c_h$, the first-best schedule satisfies the incentive compatibility constraint (IC) if and only if

$$(1-\sigma)\left[1 - \beta(1-\sigma+\eta(U_\ell'))(2\rho-1)\right] + \eta'(U_\ell' - U_h') \geq 0.$$

The first term on the LHS is positive because $\beta(1-\sigma+\eta(U))$ is strictly smaller than 1. The second term on the LHS is positive because U_ℓ' is strictly greater than U_h'. Therefore, the incentive compatibility constraint is satisfied.

For $\hat{c}^{t-1} = c^{t-1}$ and $c_t = c_h$, the first-best schedule satisfies the incentive compatibility constraint (IC) if and only if

$$\eta'(U'_\ell - U'_h) - (1-\sigma)\left[1 - \beta\left(1 - \sigma + \eta(U'_h)\right)(2\rho - 1)\right] \geq 0.$$

The derivative of the LHS with respect to the persistence ρ of productivity shocks is given by

$$\eta' \left\{ \frac{dU'_\ell}{d\rho} + \left[1 - \beta\left(1-\sigma\right)\left(2\rho-1\right)\right] \frac{dU'_h}{d\rho} \right\} + \left(1-\sigma\right)\beta 2(1 - \sigma + \eta\left(U'_h\right)).$$

If $dU'_\ell/d\rho > 0$ and $dU'_h/d\rho < 0$, the derivative is strictly positive and, hence, there exists a critical level of persistence $\rho^* \in [1/2, 1]$ such that the incentive compatibility constraint is satisfied when and only when $\rho \geq \rho^*$.

In order to identify the sign of $dU'_i/d\rho$, it is convenient to let $P_i(U_\ell, U_h; \rho)$ denote the profits of a seller that has realized the production cost c_i and has committed to providing its customers with the lifetime utility U_ℓ whenever $c_t = c_\ell$ and with U_h whenever $c_t = c_h$, i.e.

$$P_i(U_\ell, U_h; \rho) = \left(1 - \sigma + \eta(U_i)\right)\left[p_i - c_i + \beta\rho P_i + \beta(1-\rho)P_{-i}\right]$$

$$p_i(U_\ell, U_h; \rho) = u - U_i + \beta\sigma Z + \beta(1-\sigma)\left[\rho U_i + (1-\rho)U_{-i}\right].$$

For a generic couple (U_ℓ, U_h), P_i is smaller than the value function Π_i. For (U_ℓ, U_h) equal to the optimal continuation values (U'_ℓ, U'_h), P_i is equal to Π_i. The derivative of P_i with respect to the persistence of the cost of production is given by

$$\frac{\partial P_i}{\partial \rho} = \Delta^{-1}\beta\left[(1-\sigma)(U_i - U_{-i}) + (P_i - P_{-i})\right],$$

where Δ is a positive constant. When evaluated at (U'_ℓ, U'_h), $\partial P_\ell/\partial\rho$ is strictly positive and $\partial P_h/\partial\rho$ is strictly negative because $U'_\ell > U'_h$ and $P_\ell = \Pi_\ell > \Pi_h = P_h$.

The value function $\pi_i(V; \rho)$ associated to the second-stage problem in (4) is equal to

$$\pi_i(V; \rho) = u - c_i - V + \beta\sigma Z + \beta \max_{U_\ell, U_h} \textstyle\sum_j \Pr(c_j|c_i)\left[P_j(U_\ell, U_h; \rho) + (1-\sigma)U_j\right]$$

and its derivative with respect to the persistence ρ of productivity shocks is

$$\begin{aligned}\frac{\partial \pi_i}{\partial \rho} &= \beta\left[(1-\sigma)(U'_i - U'_{-i}) + (P_i - P_{-i}) + \rho\frac{\partial P_i}{\partial \rho} + (1-\rho)\frac{\partial P_{-i}}{\partial \rho}\right] = \\ &= \frac{\beta}{1-\sigma+\eta(U'_i)}\frac{\partial P_i}{\partial \rho}.\end{aligned}$$

From the second line, it follows that $\partial\pi_\ell/\partial\rho$ is strictly positive and $\partial\pi_h/\partial\rho$ is strictly negative. In turn, from the first order condition (6) for the continuation value, it follows that $dU'_\ell/d\rho$ is strictly positive and $dU'_h/d\rho$ is strictly negative.

A.5 Preliminaries to Propositions 3 and 4

Consider the first-stage problem in (11). For $V < Z$, the objective function $(1-\sigma+\eta(V)) \cdot \pi_i(V)$ is equal to zero because $\eta(V) = \sigma - 1$. For $V = Z$, the function is strictly positive because $1+\sigma = \eta(V) = 1-\sigma > 0$ and $\pi_i(Z) > 0$ as proved in Lemma 2. For $V \geq Z$, the function is quasi-concave. The function attains its maximum for $V = \underline{U}_i$, where $\underline{U}_i$ is the solution to equation (A2). From the properties of the objective function, it follows that the solution $V_i(U)$ to the first-stage problem is $\underline{U}_i$ whenever $U \leq \underline{U}_i$ and U otherwise. In turn, the value function $\Pi_i(U)$ associated to the first-stage problem is constant at $(1-\sigma+\eta(\underline{U}_i)) \cdot \pi(\underline{U}_i)$ whenever $U \leq \underline{U}_i$ and is strictly decreasing and quasi-concave otherwise.

Next, consider the second-stage problem in (11). As proved in Lemma 2, the choice set can be restricted to the continuation values (U'_ℓ, U'_h) that are greater than the profit-maximizing values $(\underline{U}_\ell, \underline{U}_h)$. Over this domain, the objective function is jointly quasi concave in (U'_ℓ, U'_h). Also, for any given U'_h, the objective function is maximized at U'^*_ℓ, where U'^*_ℓ is the solution to equation (A3) for $j = \ell$ and is strictly greater than $\underline{U}_\ell$. For any given U'_ℓ, the objective function is maximized at U'^*_h, where U'^*_h is the solution to equation (A3) for $j = h$ and is strictly greater than $\underline{U}_h$. The choice of the continuation values is limited by the incentive compatibility constraint $\Pi_j(U'_j) \geq \widetilde{\Pi}_{-j}(U'_{-j})$. Since V enters separately from (U'_ℓ, U'_h) in the objective function and does not enter the constraints, the solution $U'_{j|i}(V)$ to the second-stage problem is independent from V and can be denoted with $U'_{j|i}$.

A.6 Proof of Proposition 3

When $\rho = 1/2$, the second-stage problem in (12) can be reformulated as

$$\pi_i(V) = u - c_i - V + \beta\sigma Z + \frac{\beta}{2} \max_{U'_j \geq \underline{U}_j} \textstyle\sum_j \left[\Pi_j(U'_j) + (1-\sigma)U'_j\right], \text{ s.t.}$$

$$(1-\sigma+\eta(U'_j)) \cdot \pi_j(U'_j) \geq (1-\sigma+\eta(U'_{-j})) \cdot \pi_j(U'_{-j}). \tag{A6}$$

Since $U'_h \geq \underline{U}_h$, $U'_\ell \geq \underline{U}_\ell \geq \underline{U}_h$ and the function $(1-\sigma+\eta) \cdot \pi_h$ is strictly decreasing for all $U \geq \underline{U}_h$, the high-cost seller's incentive compatibility constraint (A6) is equivalent to $U'_\ell \geq U'_h$. Since the function $(1-\sigma+\eta) \cdot \pi_\ell$ is strictly increasing for $U \in [\underline{U}_h, \underline{U}_\ell]$ and strictly decreasing for $U \geq \underline{U}_\ell$, the low-cost seller's incentive compatibility constraint (A6) is satisfied either if $U'_h \geq U'_\ell$ or if U'_h is sufficiently lower than $\underline{U}_\ell$. Overall, a couple of continuation values (U'_ℓ, U'_h) is feasible if either $U'_\ell = U'_h \geq \underline{U}_\ell$ or $U'_\ell \neq U'_h$ and $U'_h \leq \underline{U}_\ell$, $U'_\ell \geq \underline{U}_\ell$.

Let $c_\ell(\Delta) = c - \Delta$ and $c_h(\Delta) = c + \Delta$ for some $c \in (0, u-z)$ and $\Delta \geq 0$. If the solution to the second-stage problem is such that $U'_\ell = U'_h$, the seller's profits per customer are bounded below by

$$\pi_i^{\mathcal{P}}(V;\Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \frac{\beta}{2}\sum_j \left[\Pi_j\left(U_\ell'^*(\Delta);\Delta\right) + (1-\sigma)\,U_\ell'^*(\Delta)\right].$$

If the solution to the second-stage problem is such that $U'_\ell \neq U'_h$, the seller's profits per customer are bounded above by

$$\pi_i^{\mathcal{S}}(V;\Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \frac{\beta}{2}\sum_j \left[\Pi_j(U_j^{\mathcal{S}}(\Delta);\Delta) + (1-\sigma)\,U_j^{\mathcal{S}}(\Delta)\right],$$

where $U_h^{\mathcal{S}}(\Delta) = \min\{U_h'^*(\Delta), \underline{U}_\ell(\Delta)\}$ and $U_\ell^{\mathcal{S}}(\Delta) = U_\ell'^*(\Delta)$. Independently from the nature of the solution to the second-stage problem, the seller's profits per customer are bounded above by

$$\pi_i^{\mathcal{P}}(V;\Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \frac{\beta}{2}\sum_j \left[\Pi_j(U_j'^*(\Delta);\Delta) + (1-\sigma)\,U_j'^*(\Delta)\right].$$

For $\Delta = 0$, $\pi_i^{\mathcal{P}}(V;\Delta)$ is equal to $\pi_i^*(U;\Delta)$ because $\Pi_\ell(U;\Delta) = \Pi_h(U;\Delta)$ and $U_\ell'^*(\Delta) = U_h'^*(\Delta)$. For $\Delta = 0$, $\pi_i^{\mathcal{S}}(U;\Delta)$ is strictly smaller than to $\pi_i^*(U;\Delta)$ because $U_h^{\mathcal{S}}(\Delta) = \underline{U}_h(\Delta)$ and $\underline{U}_h(\Delta) < U_h'^*(\Delta)$. By continuity, I conclude that the solution to the second-stage problem is such that $U'_\ell = U'_h$ for all $\Delta \in (0, \Delta^*)$.

A.7 Proof of Proposition 4

In order to characterize the second-best price schedule when productivity shocks are persistent, I start by conjecturing that the solution to the problem (SP2) is such that: (i) the high-cost seller's incentive compatibility constraint is moot, i.e. $\Pi_h(U'_{h|i}) > \tilde{\Pi}_\ell(U'_{\ell|i})$ for $i = \ell, h$; (ii) the low-cost seller prefers to report its true type rather than lying whenever $U'_\ell = U'_h = U$, i.e. $\pi_\ell(U) \geq \widetilde{\pi}_h(U)$.

If the high-cost seller's incentive compatibility constraint is moot, the second-stage problem in (12) can be reformulated as

$$\pi_i(V) = u - c_i - V + \beta\sigma Z + \max_{U'_j \geq \underline{U}_j} \sum_j \Pr(c_j|c_i)\left[\Pi_j(U'_j) + (1-\sigma)U'_j\right], \text{ s.t.}$$

$$(1 - \sigma + \eta(U'_\ell)) \cdot \pi_\ell(U'_\ell) \geq (1 - \sigma + \eta(U'_h)) \cdot \widetilde{\pi}_h(U'_h). \tag{A7}$$

For all $U'_\ell \geq \underline{U}_\ell$, the LHS of (A7) is strictly decreasing. For all $U'_h \geq \underline{U}_h$, the RHS of (A7) is quasi-concave because it is concave whenever increasing and convex only when strictly decreasing. The RHS attains its maximum for $U'_h = \widetilde{\underline{U}}_h \leq \underline{U}_\ell$, where $\widetilde{\underline{U}}_h$ satisfies

$$\left[\eta'(\widetilde{\underline{U}}_h) \cdot \widetilde{\pi}_h(\widetilde{\underline{U}}_h) - (1 - \sigma + \eta(\widetilde{\underline{U}}_h))\right] \cdot (\widetilde{\underline{U}}_h - Z) = 0.$$

It is useful to partition the feasible set of the second-stage problem into the subsets $\mathcal{P}$ and $\mathcal{S}$. Specifically, $\mathcal{P}$ is the set of continuation values (U'_ℓ, U'_h) that are feasible and such that $U'_h \geq \widetilde{\underline{U}}_h$, while $\mathcal{S}$ is the set of continuation values that are feasible and such that $U'_h \leq \widetilde{\underline{U}}_h$. The set $\mathcal{P}$ contains the state-independent continuation values $U'_\ell = U'_h \geq \underline{U}_\ell$ because $\pi_\ell(U') \geq \widetilde{\pi}_h(U')$. The set $\mathcal{S}$ does not contain any continuation values (U'_ℓ, U'_h) such that U'_h is greater than $\underline{U}_\ell$ because $\widetilde{\underline{U}}_h \leq \underline{U}_\ell$.

Let $c_\ell(\Delta) = c - \Delta$ and $c_h(\Delta) = c + \Delta$ for some $c \in (0, u - z)$ and $\Delta \geq 0$. If the solution to the second-stage problem belongs to the subset $\mathcal{P}$, the seller's profits per customer are bounded below by

$$\pi_i^{\mathcal{P}}(V;\Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[\Pi_j\left(U_\ell'^*(\Delta);\Delta\right) + (1-\sigma)\, U_\ell'^*(\Delta)\right].$$

If the solution to the second-stage problem belongs to the subset $\mathcal{S}$, the seller's profits per customer are bounded above by

$$\pi_i^{\mathcal{S}}(V;\Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[\Pi_j(U_j^{\mathcal{S}}(\Delta);\Delta) + (1-\sigma)\, U_j^{\mathcal{S}}(\Delta)\right],$$

where $U_h^{\mathcal{S}}(\Delta) = \min\{U_h'^*(\Delta), \underline{U}_\ell(\Delta)\}$ and $U_\ell^{\mathcal{S}}(\Delta) = U_\ell'^*(\Delta)$. Independently from the nature of the solution to the second-stage problem, the seller's profits per customer are bounded above by

$$\pi_i^{\mathcal{P}}(V;\Delta) = u - c_i(\Delta) - V + \beta\sigma Z + \beta \textstyle\sum_j \Pr(c_j|c_i)\left[\Pi_j(U_j'^*(\Delta);\Delta) + (1-\sigma)\, U_j'^*(\Delta)\right].$$

For $\Delta = 0$, $\pi_i^{\mathcal{P}}(V;\Delta)$ is equal to $\pi_i^*(U;\Delta)$ because $\Pi_\ell(U;\Delta) = \Pi_h(U;\Delta)$ and $U_\ell'^*(\Delta) = U_h'^*(\Delta)$. For $\Delta = 0$, $\pi_i^{\mathcal{S}}(U;\Delta)$ is strictly smaller than to $\pi_i^*(U;\Delta)$ because $U_h^{\mathcal{S}}(\Delta) = \underline{U}_h(\Delta)$ and $\underline{U}_h(\Delta) < U_h'^*(\Delta)$. By continuity, I conclude that the solution to the second-stage problem belongs to $\mathcal{P}$ for all $\Delta \in (0, \Delta_1)$.

When Δ is sufficiently small, i.e. $\Delta \in (0, \Delta_2)$, the unconstrained maximum of the second-stage problem $(U_\ell'^*, U_h'^*)$ is not feasible because it violates the low-cost seller's incentive compatibility constraint (cf. condition (10)). When this is the case, the constraint (A7) holds with equality because the objective function of the second-stage problem is quasi concave. Therefore, for all $\Delta \in (0, \Delta^*)$, where $\Delta^* = \min\{\Delta_1, \Delta_2\}$, the solution to the second-stage problem belongs to the subset $\mathcal{P}$ and satisfies the constraint (A7) with equality.

For all $\Delta \in (0, \Delta^*)$, the solution $(U'_{\ell|i}, U'_{h|i})$ to the second-stage problem has the following properties:

1. The continuation value $U'_{\ell|i}$ is smaller than $U_\ell'^*$. Proof: If $U'_{\ell|i}$ is strictly greater than $U_\ell'^*$, then $(U_\ell'^*, U'_{h|i})$ is feasible because $\Pi_\ell(U_\ell'^*) > \Pi_\ell(U'_{\ell|i})$. Also, $(U_\ell'^*, U'_{h|i})$ is preferable because the objective function is quasi concave in U'_ℓ and is maximized at $U_\ell'^*$.

2. The continuation value $U'_{h|i}$ is greater than U'^*_h. Proof: If $U'_{h|i}$ is strictly smaller than U'^*_h, then $(U'_{\ell|i}, U'^*_h)$ is feasible because $\tilde{\Pi}_h(U'^*_h) < \tilde{\Pi}_h(U'_{h|i})$. Also, $(U'_{\ell|i}, U'^*_h)$ is preferable because the objective function is quasi concave in U'_h and is maximized at U'^*_h.

3. The continuation value $U'_{h|i}$ is strictly smaller than $U'_{\ell|i}$. Proof: If $U'_{h|i}$ is greater than $U'_{\ell|i}$, then $\Pi_\ell(U'_{\ell|i}) \geq \tilde{\Pi}_h(U'_{\ell|i}) > \tilde{\Pi}_h(U'_{h|i})$. This is not possible because (A7) holds with equality for all $\Delta \in (0, \Delta^*)$.

4. If $U'_{h|j} > U'_{h|i}$, then $U'_{\ell|j}$ is strictly greater than $U'_{\ell|i}$. Proof: Since the LHS and RHS of (A7) are strictly decreasing in U'_ℓ and U'_h and the constraint (A7) holds with equality, if $U'_{h|j} > U'_{h|i}$ then $U'_{\ell|j} > U'_{\ell|i}$.

5. The continuation values are such that $U'_{\ell|\ell} \geq U'_{\ell|h}$ and $U'_{h|\ell} \geq U'_{h|h}$. Proof: Since the objective function puts more weight on $\Pi_\ell(U'_\ell) + (1-\sigma)\, U'_\ell$ and less weight on $\Pi_h(U'_h) + (1-\sigma)\, U'_h$ when $c_i = c_\ell$ than when $c_i = c_h$, $\Pi_\ell(U'_{\ell|\ell}) + (1-\sigma)\, U'_{\ell|\ell}$ is greater than $\Pi_\ell(U'_{\ell|h}) + (1-\sigma)\, U'_{\ell|h}$. In light of property (1), this implies that $U'_{\ell|\ell}$ is greater than $U'_{\ell|h}$. In light of property (2), this implies that $U'_{h|\ell}$ is greater than $U'_{h|h}$.

6. The continuation values are such that $E\left[(1-\sigma)U'_{j|i} + \Pi_i(U'_{j|i})|c_k\right]$ is greater for $k = i$ than $-i$. Proof: This result follows immediately from property (5).

These six properties of the optimal continuation values lead immediately to Proposition 4.

In the last step of the analysis, I have to verify my initial conjectures. In order to verify that the high-cost seller's incentive compatibility constraint is moot, it is convenient to rewrite $\Pi_h(U'_{h|i}) \geq \tilde{\Pi}_\ell(U'_{\ell|i})$ as

$$\eta'(U'_{h|i} - U'_{\ell|i}) \cdot \pi_h(U'_{h|i}) \geq$$

$$(1-\sigma+\eta(U'_{\ell|i})) \left[\begin{array}{l} U'_{h|i} - U'_{\ell|i} + \beta\,(1-\sigma)\,(2\rho-1)\,(U'_{\ell|\ell} - U'_{h|\ell}) + \\ \\ \beta \left\{ E\left[(1-\sigma)U'_{i|\ell} + \Pi_i(U'_{i|\ell})|c_h\right] - E\left[(1-\sigma)U'_{i|h} + \Pi_i(U'_{i|h})|c_h\right] \right\}. \end{array} \right] \tag{A8}$$

First, notice that $U'_{h|i} \geq U'^*_h$ implies that $\eta'\pi_h(U'_{h|i})$ is smaller than $\eta(U'_{h|i})$ and that the LHS of (A8) is bounded below by

$$(U'_{h|i} - U'_{\ell|i}) \cdot \eta(U'_{h|i}). \tag{A9}$$

Secondly, notice that $E\left[(1-\sigma)U'_{i|h} + \Pi_i(U'_{i|h})|c_h\right]$ greater than $E\left[(1-\sigma)U'_{i|\ell} + \Pi_i(U'_{i|\ell})|c_h\right]$ and $U'_{\ell|h} - U'_{h|h}$ greater than $U'_{\ell|\ell} - U'_{h|\ell}$ (a fact that can be derived from the low-cost seller's incentive compatibility constraint) imply that the RHS of (A8) is bounded above by both

$$\begin{aligned}&(1-\sigma+\eta(U'_{\ell|i}))\left[U'_{h|i}-U'_{\ell|i}+\beta\,(1-\sigma)\,(2\rho-1)\,(U'_{\ell|\ell}-U'_{h|\ell})\right],\\&(1-\sigma+\eta(U'_{\ell|i}))\left[U'_{h|i}-U'_{\ell|i}+\beta\,(1-\sigma)\,(2\rho-1)\,(U'_{\ell|h}-U'_{h|h})\right].\end{aligned} \tag{A10}$$

Overall, the high-cost seller's incentive constraint (A8) is satisfied if (A9) is greater than (A10) or, equivalently, if

$$(1-\sigma)\left[1-\beta(1-\sigma+\eta(U'_{h|i}))(2\rho-1)\right]+\eta'(U'_{\ell|i}-U'_{h|i})\geq 0. \tag{A11}$$

Because $\beta(1-\sigma+\eta(U'_{h|i}))$ is smaller than 1 and $U'_{\ell|i}$ is greater than $U'_{h|i}$, the sufficient condition (A11) is satisfied.

Finally, I have to verify the conjecture that the low-cost seller prefers to report its true type rather than lying whenever $U'_\ell = U'_h = U$, i.e. $\pi_\ell(U) \geq \tilde{\pi}_h(U)$ or

$$\begin{aligned}&E\left[\Pi_j(U'_{j|\ell})+(1-\sigma)\,U'_{j|\ell}|c_\ell\right]\geq\\&E\left[\Pi_j(U'_{j|h})+(1-\sigma)\,U'_{j|h}|c_\ell\right]+(2\rho-1)\,(1-\sigma)\,(U'_{h|h}-U'_{\ell|h}).\end{aligned} \tag{A12}$$

Since $E\left[\Pi_j(U'_{j|\ell})+(1-\sigma)\,U'_{j|\ell}|c_\ell\right]$ is greater than $E\left[\Pi_j(U'_{j|h})+(1-\sigma)\,U'_{j|h}|c_\ell\right]$ and $U'_{h|h}$ is smaller than $U'_{\ell|h}$, condition (A12) is satisfied.