Variable Search Intensity in an Economy with Coordination Unemployment

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Abstract

This paper analyzes an urn–ball matching model in which workers decide how intensively they sample job openings and apply at a stochastic number of suitable vacancies. Equilibrium is not constrained efficient; there is excessive job creation and search intensity can be too high or too low. Moreover, an inefficient discouraged–worker effect among homogenous workers emerges under adverse labor market conditions. The model is then calibrated to the U.S. economy. Unlike previous coordination–friction economies with fixed search intensity, the model with variable search intensity is quantitatively consistent with the relations between unemployment, vacancies and the job–finding rate. It also exhibits an amplification of productivity shocks which is of the same magnitude as in the data.

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1 Introduction

Search and matching models are widely used to address various labor market phenomena, such as unemployment, worker and job flows, and wage dispersion.[1] A large portion of the literature utilizes the idea of an aggregate matching function which maps the stocks of searching workers and firms into the flow of new matches. Despite of its modeling advantages, the reduced–form matching function suffers from two drawbacks. One is its inability to deal with heterogeneity convincingly. Of course, the foremost purpose of this function is to abstract from any explicit source of frictions (including heterogeneity) to describe the implications of costly trading in the labor market with a minimum amount of complexity. Yet, many important issues (for example, the pattern of skill premia) require an explicit analysis of how heterogeneity affects labor market outcomes.[2] The other drawback is that a reduced–form matching function is, by construction, invariant to policy. Again a more explicit model of frictions is needed to answer the question how policy affects the matching relationship (see also Lagos (2000) and Shimer (2007a)).

There is a large literature on microeconomic foundations behind the aggregate matching function; see section 3 of Petrongolo and Pissarides (2001) for a survey. One such foundation rests on coordination frictions; early contributions are Butters (1977), Hall (1977) and Montgomery (1991), more recent ones are Burdett, Shi, and Wright (2001), Julien, Kennes and King (2000, 2006) and Albrecht, Gautier, and Vroman (2006). The key idea is simple: since workers do not coordinate their application decisions and firms do not coordinate their job–offer decisions, some firms end up with no applications while others get many, and some workers obtain several job offers while others have none. So at the end of every period, unfilled jobs and unemployed workers coexist. These models give rise to well–behaved aggregate

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[1]See Rogerson, Shimer, and Wright (2005) for a survey. From a quantitative perspective, search and matching models have been successful in describing the relation between unemployment, vacancies and the worker flows between employment and unemployment (see e.g. Blanchard and Diamond (1989)). They are less successful in generating the observed magnitude of fluctuations of the vacancy–unemployment ratio (Shimer (2005)) and the observed amount of wage dispersion (Hornstein, Krusell, and Violante (2006)).

[2]There is a number of papers utilizing reduced–form matching functions in models with heterogenous jobs or heterogenous skills (e.g. Acemoglu (2001) and Albrecht and Vroman (2002)). But these models must rest on ad–hoc assumptions on how workers and jobs of different types are matched.

matching functions which typically have constant returns in economies with a large number of workers and jobs.

Still there are at least two open issues with coordination–friction models. The first is that search intensity is typically constant. Although it is straightforward to include variable search intensity in standard search models with exogenous matching functions (see Chapter 5 of Pissarides (2000)), it is a less obvious matter in economies where matching frictions result explicitly from coordination problems. An exception is the model of Albrecht, Gautier, and Vroman (2006) where workers send multiple applications; in Section 5.1 of their paper they discuss what happens if applications are costly and workers decide about the optimal number. However, because of the discrete–choice nature of this problem, there can be multiple equilibria and the problem is analytically intractable. Also, it is not clear if the number of applications is an appropriate measure for "search intensity". Chance plays an important role in the search for jobs; some workers who search hard may simply be unlucky, find few suitable job openings and send few applications. Others who spend less time on search, may notice a larger number of adequate job openings and send more applications. The second open issue is quantitative: can coordination–friction models, when reasonably calibrated to long–run averages, match the Beveridge curve and the relation between the vacancy–unemployment ratio and the job–finding rate?[3]

This paper is an attempt to fill these gaps. The first contribution of the paper is theoretical. I analyze an urn–ball matching model in which workers decide about the rate at which they sample job openings ("search intensity") and apply at all suitable jobs they observe. For a given search intensity, the actual number of suitable jobs (and so the number of applications) is stochastic. The *expected* number of applications, however, increases proportionately with search intensity. As applications are sent randomly, wages are determined by ex–post competition, according to the same bidding game as in Julien, Kennes and King (2000, 2006) and Albrecht, Gautier, and Vroman (2006). Workers with at least two offers receive the competitive wage, those who have only one offer are paid the reservation wage. A key advantage

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[3]Recently, Mortensen (2007) and Shimer (2007a) have analyzed microfoundations of the matching function which are based on mismatch (instead of coordination frictions) and which generate striking quantitative results along these dimensions. Julien, Kennes, and King (2006) examine a coordination–friction model quantitatively, but their focus is wage dispersion, and not the Beveridge curve and the elasticity of the matching function.

of my model is that search intensity is a continuous choice variable, which makes the model analytically tractable and allows for an explicit equilibrium characterization using first–order conditions. When labor market conditions are good, there are many job openings per worker and all workers search with the same intensity. With less favorable conditions, however, there are fewer job openings and there is no symmetric equilibrium in pure search–intensity strategies. Instead, some workers are active and search with a common positive intensity, while others remain inactive and decide not to search at all. Thus, the model describes endogenous nonparticipation in an environment where all workers are equally productive and have the same taste for leisure. When comparing these equilibrium outcomes to the choice of a social planner, I obtain the following results: (i) nonparticipation is never constrained efficient; (ii) entry is always excessive, for the same reason as in Albrecht, Gautier, and Vroman (2006), and (iii) search intensity can be too high or too low.

The second contribution of this paper is quantitative. In Section 3.5 I show that existing coordination–friction models with fixed search intensity, when properly calibrated with a reasonable choice of the period length, are unable to account for the observed slope of the Beveridge curve in the U.S. since 2001 and for the empirical elasticity of the matching function. Specifically, the Beveridge curve is much too steep and the job–finding rate responds too little to variations in the vacancy–unemployment ratio. In contrast, my model with variable search intensity and with a reasonable choice of the intertemporal elasticity of substitution of leisure, performs reasonably well along these dimensions. The Beveridge curve becomes flatter and the elasticity of the job–finding rate increases substantially. Another finding is that the model exhibits an amplification of productivity shocks which is about 10 times as large as in Shimer's (2005) calibration of the search and matching model. About half of this difference is due to variable search intensity. The explanation for the other half is that unemployment utility is much larger here than in Shimer's calibration; utility of the unemployed is decomposed into an income component and a leisure component. The income component matches the benefit replacement ratio, the leisure component follows from deliberate (and cautious) choices of leisure time for employed and unemployed workers. The large value of unemployment utility here is similar as in the calibration of the search and matching model of Hagedorn and Manovskii (2007), though the calibration strategy is quite different from theirs.

Section 2 of the paper develops the theoretical model in a static setting. Section

3 embeds the static model in a dynamic environment and analyzes its quantitative implications, both positively and normatively. Section 4 concludes.

2 The static model

2.1 The setup

Consider a one–period economy with a large number M of identical workers and a large number of N of identical firms, each creating one vacancy. The number of workers is fixed, but the number of firms is determined from a free–entry condition. I consider the limit where both M and N tend to infinity and where $q = M/N$, the number of workers per job opening, is positive and finite. All agents are risk neutral and aim to maximize their expected income. At the end of the period, unemployment income is zero and employed workers produce p units of output (=job surplus). I consider the following sequence of events.

Stage I Firms enter at marginal cost $c(1/q)$ where c is a weakly increasing function of the number of job openings per worker.

Stage II Every worker decides search intensity λ at cost $k(\lambda)$, where k satisfies $k'(\lambda) > 0$ and $k''(\lambda) \geq 0$ for all $\lambda \geq 0$. If worker i searches with intensity λ, he observes a suitable vacancy at firm j with probability λ/N and applies there. These stochastic events are independent across i and j.

Stage III Each firm makes a wage offer to at most one applicant, rejecting all others.

Stage IV Workers credibly reveal to firms how many offers they have, and firms can simultaneously revise their initial bids.

Stage V Workers decide what offer (if any) to accept.

I impose the usual anonymity restriction that every worker treats all (identical) firms equally (at stage V) and that every firm treats all workers equally (at stage III).

Two remarks are in order. First, the specification that marginal entry costs are not constant is needed to limit entry activity in an equilibrium where some workers are discouraged. The assumption can be justified, for example, by non–labor inputs in fixed supply (e.g. land) whose prices increase in the number of active firms. Alternatively, firms may differ in their setup costs, or the cost of a job advertisement can increase with the total number of job openings. Second, search intensity is a continuous variable which determines the likelihood λ/N that a worker observes a suitable job opening at any firm. This likelihood is plausibly proportional to $1/N$: the worker samples a certain (random) segment of the labor market whose size increases with search intensity λ. If the number of firms becomes larger, the size of the sampled segment stays the same, but the probability that a given firm belongs to this segment falls with factor $1/N$.

In the large economy, the number of applications (per worker and per job) are Poisson distributed. A worker with search intensity λ applies at exactly n firms with probability

$$\binom{N}{n}\left(\frac{\lambda}{N}\right)^n\left(\frac{N-\lambda}{N}\right)^{N-n} \approx \frac{1}{n!}\lambda^n e^{-\lambda} \ .$$

Conversely, if all workers search with intensity λ, a firm receives applications from exactly m workers with probability

$$\binom{M}{m}\left(\frac{\lambda}{N}\right)^m\left(\frac{N-\lambda}{N}\right)^{M-m} \approx \frac{1}{m!}(\lambda q)^m e^{-\lambda q} \ .$$

Thus the expected number of applications per worker is λ and the expected number of applications per firm is λq.

The last three stages of the above game have the following solution. Firms offer the reservation wage at stage III, revising the offer at stage IV only if the worker reveals another offer, in which case Bertrand competition drives wage offers to the marginal product. At the last stage, anonymity implies that workers randomize between equal offers. In this respect, my model resembles those of Julien, Kennes, and King (2000) and Albrecht, Gautier, and Vroman (2006) where workers with only one offers are paid the monopsony wage and workers with multiple offers receive the competitive wage. The setting of Julien et al. is the limiting case of my model where $k(.) = 0$ and $\lambda/N = 1$, so every worker applies at all jobs. With $\lambda < \infty$, however, workers send finitely many applications in expectation. In the model of Albrecht et al. all workers send the same number of applications. Here, in contrast, the number of applications is stochastic, reflecting the role of chance in the search process. Workers do not

decide at how many firms they apply, but rather how intensively they sample job openings. The model of Albrecht et al. also has an (irrelevant) wage posting stage prior to the application stage where firms commit to a lower wage bound which happens to coincide with the reservation wage. In this model such a wage–posting stage makes little sense since applications are sent randomly.[4]

2.2   The matching function

Before solving the model, it is useful to consider the matching function of this model. Assume, for the time being, that all workers decide the same search intensity λ at stage II. For any worker i the probability to get an offer from firm j, conditional on i applying at j, is[5]

$$z \equiv \frac{1 - e^{-\lambda q}}{\lambda q} \ . \tag{1}$$

Hence, for any worker the probability to receive at least one offer (and thus to find a job) is

$$\sum_{n \geq 1} \frac{1}{n!} \lambda^n e^{-\lambda} \left[1 - (1 - z)^n \right] = 1 - e^{-\lambda z} = 1 - e^{-\frac{1 - e^{-\lambda q}}{q}} \equiv m(q, \lambda) \ . \tag{2}$$

The matching rate for workers is declining in q (as usual) and strictly increasing in λ: the more applications workers send on average, the more likely it is that every worker receives an offer. Such a result is not obvious; in fact it does not hold in the model of Albrecht, Gautier, and Vroman (2006) where the matching rate can be *declining* in the fixed (non–stochastic) number of applications. The reason for their result is that there are two coordination frictions with multiple applications. The first friction is based on lack of coordination between workers: some firms receive no applications while others receive multiple applications since workers do not coordinate at the

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[4]Although there are no posted wages, this model falls into the class of "directed search" models since both firms and workers direct their contacts after the application stage: firms decide what worker type to approach at stage III, and workers decide what type of job to accept at stage V. Of course, these issues are irrelevant in this model with identical agents, but they play a role when workers or jobs are heterogenous. Moreover, an extension to heterogenous job types can also yield a directed search aspect at stage II if workers decide how intensively they sample jobs of different types.

[5]The derivation is standard: Prob(i gets offer from j | i applies at j)Prob(i applies at j)= $z \cdot \lambda / N$ is equal to Prob(j gets ≥ 1 appl.)Prob(i gets offer from j | j gets ≥ 1 appl.)= $(1 - e^{-\lambda q}) \cdot 1/M$. Solving yields z.

application stage. The second friction is due to a lack of coordination between firms at the job offer stage: some workers do not receive any offer, others have multiple offers. Raising the number of applications mitigates the first friction but aggravates the second one: it becomes more likely that multiple firms contact the same worker. In my model the first effect always dominates so that the number of matches is globally increasing in the common search intensity.

When $\lambda \to \infty$, workers apply at all firms at stage I and the matching function is $m_J(q) \equiv 1 - e^{-1/q}$, the same as in the model of Julien, Kennes, and King (2000). In this limit only the second coordination friction is at work. In the model of Albrecht et al. (2006), the matching function is $m_A(q, a) \equiv 1 - [1 - (1 - e^{-aq})/(aq)]^a$ when all workers send a applications; again the matching function of Julien et al. emerges as the special case $a \to \infty$. For finite a, it may be that $m_A(q, a) > m_J(q)$, so matching is more efficient with fewer applications. In my model, in contrast, matching is always more efficient the more applications are sent, i.e. $m(q, \lambda) < m_J(q)$ holds for finite λ. It can also be shown that $m(q, \lambda) < m_A(q, \lambda)$; matching is more efficient when all workers send the same number of applications $a = \lambda$ than when they randomize applications from a Poisson distribution with mean λ.

2.3 Equilibrium search intensity

Consider the search intensity decision of workers at stage II after firm entry, so the worker–job ratio q is given. As discussed before, a worker obtains income p if he receives two or more offers at stage III, but he ends up with zero income otherwise. The probability to have two or more offers, conditional on n applications, is

$$1 - (1 - z)^n - nz(1 - z)^{n-1} \ .$$

Hence, when the worker's search intensity is λ, the probability to end up with at least two offers is

$$\sum_{n \geq 2} \frac{1}{n!} \lambda^n e^{-\lambda} \Big[1 - (1 - z)^n - nz(1 - z)^{n-1} \Big] = 1 - e^{-\lambda z}(1 + \lambda z) \ .$$

When an individual worker in a large market decides λ, he takes z (the probability to get an offer, conditional on applying) as given. Hence, every worker solves

$$\max_{\lambda \geq 0} U_z(\lambda) \equiv \Big[1 - e^{-\lambda z}(1 + \lambda z) \Big] p - k(\lambda) \ .$$

This objective function is typically not concave; for many cost functions (e.g. linear) it is convex at low values of λ and concave at higher values. Moreover, $\lambda = 0$ is always a local maximum with $U_z(0) = 0$ and $U_z'(0) = -k'(0) < 0$. If there is another local maximum, it satisfies the first–order condition $U_z'(\lambda) = 0$, which is

$$k'(\lambda) = \lambda z^2 e^{-\lambda z} p . \tag{3}$$

The right–hand side of this equation is hump–shaped and the left–hand side is weakly increasing. For large enough productivity, this equation has two solutions, of which the smaller one is a local minimum and the larger is a local maximum.

Consider first a *pure–strategy equilibrium* where all workers choose the same search intensity $\lambda^* > 0$. Combining (1) and (3) shows that λ^* solves

$$k'(\lambda) = \frac{(1 - e^{-\lambda q})^2 e^{-\frac{1-e^{-\lambda q}}{q}}}{\lambda q^2} p . \tag{4}$$

The right–hand side of this equation is hump–shaped in λ and it equals zero at $\lambda = 0$ and at $\lambda = \infty$. Again, for p large enough, there are typically two solutions of this equation. The smaller solution is irrelevant since it gives rise to a local minimum of workers' objective function. The larger solution corresponds to a local maximum of U_z which constitutes a pure–strategy equilibrium, provided that workers do not want to deviate to the local maximum at $\lambda = 0$, i.e. it must hold that $U_z(\lambda^*) \geq 0$. If this condition is not satisfied, the unique symmetric equilibrium must have mixed strategies where workers end up searching with different intensities.

To obtain an analytical result, Proposition 1 characterizes existence of an equilibrium in pure strategies in the special case of linear search costs.

Proposition 1: *Consider the search cost function $k(\lambda) = k_0 \lambda$ and suppose that $p/k_0 > e^x / x$ and*
$$q \leq \bar{q} \equiv \Phi\Big(\frac{k_0 e^x}{px}\Big) \frac{k_0 e^x}{px^2} ,$$
where $\Phi(z) = \varphi$ is the inverse of $z = (1 - e^{-\varphi})/\varphi$, and $x \approx 1.79328$ is the unique positive solution of $e^x = 1 + x + x^2$. Then there exists an equilibrium of the stage II subgame where all workers search with the same intensity λ^* which is the larger solution of equation (4).

Proof: Appendix.

When there are sufficiently many jobs per worker ($q \leq \bar{q}$), all workers decide to search with the same intensity. When this condition is violated, however, some workers cease to search at all. Indeed, the symmetric equilibrium becomes one in *mixed strategies*, where some "active" workers search with positive intensity λ_A whilst others remain "inactive", deciding $\lambda = 0$. Let α denote the probability to play the active search strategy.[6] All workers must be indifferent between these strategies, so $U_z(\lambda_A) = 0$ holds. This requirement together with the first–order condition $U'_z(\lambda_A) = 0$ determine the job–offer probability z and search intensity for active workers λ_A. Therefore, these two numbers depend on productivity p and on the search cost function, but they are independent of market tightness. On the other hand, z is related to the average search intensity $\lambda = \alpha \lambda^A$ according to equation (1).[7] As in Proposition 1, let $\lambda q = \Phi(z)$ be the inverse of this relation. Thus, the fraction of active searchers is $\alpha = \Phi(z)/(q\lambda_A)$, which also shows that the number of active searchers per job αq is independent of market tightness $1/q$. Put differently, any increase in job creation triggers a proportional increase in search activity.

In the special case of a linear search cost function $k(\lambda) = k_0 \lambda$, the following analytical expressions are easily obtained:

$$\lambda_A = \frac{px^2 e^x}{k_0} \ , \quad z = \frac{k_0 e^x}{px} \ , \quad \alpha = \Phi\left(\frac{k_0 e^x}{px}\right) \frac{k_0 e^x}{px^2} \frac{1}{q} \ , \tag{5}$$

with constant x as in Proposition 1. In this case the conditions for a mixed strategy equilibrium to exist are that $\alpha > 0$ (which needs $p/k_0 > e^x/x$) and $\alpha < 1$ (which needs $q > \bar{q}$). These results can be summarized as follows.

Proposition 2:

(a) *Consider the search cost function $k(\lambda) = k_0 \lambda$ and suppose that $p/k_0 > e^x/x$ and $q > \bar{q}$. Then there exists an equilibrium of the stage II subgame where fraction $\alpha \in (0, 1)$ of workers are active with search intensity λ_A and fraction $1 - \alpha$ of workers are inactive.*

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[6]Clearly, there is another interpretation of this outcome as an asymmetric pure–strategy equilibrium where fraction α of the population is active and the remaining fraction is inactive.

[7]Note that $\alpha \lambda_A/N = \lambda/N$ is the probability that a given worker i applies at a given firm j. Hence λ is also the expected number of applications per worker and $z = (1 - e^{-\lambda q})/(\lambda q)$ is the probability of an offer, conditional on applying. The proof is the same as in footnote 5.

(b) *For an arbitrary search cost function, search intensity of active workers λ_A, the job–offer probability z and the ratio of active workers per job αq are all independent of market tightness $1/q$ in a mixed–strategy equilibrium. The job–finding probability is $\alpha(1 - e^{-\lambda_A z})$, which increases proportionately with market tightness.*

Although the existence results in Proposition 1 and 2(a) are derived for linear search costs, I conjecture that results do not change qualitatively for an arbitrary convex cost function: all workers are active with the same search intensity when the labor market is tight (small q), whilst some workers are "discouraged" when labor market prospects are less favorable from workers' perspective (large q). This conjecture has been confirmed in several numerical examples, including the parameterization used in Section 3. So in the following $\bar{q}$ denotes the threshold value of the worker–firm ratio separating an equilibrium with inactive workers from one without.

2.4 Free entry

To determine the endogenous number of jobs, note that a firm's profit is p whenever it has at least one applicant *and* when the chosen applicant has no other offer. Otherwise profit is zero. When sufficiently many firms enter, there are no inactive workers and all workers search with the same intensity $\lambda^*(q)$. Expected profit is then

$$\pi(q) = \left[1 - e^{-q\lambda^*(q)}\right] e^{-\frac{1 - e^{-q\lambda^*(q)}}{q}} p \ , \ q \leq \bar{q} \ . \tag{6}$$

The expression in squared brackets is the probability that the firm has at least one applicant, and the second term is the probability that a randomly chosen applicant has no other offer. For fixed λ^*, profit is strictly increasing in q: the larger the worker–job ratio, the more likely it is that a firm finds an applicant and the less likely it is that an applicant has multiple offers. When the effect of q on λ^* is taken into account, the overall impact of q on π is more complex since both the effect of q on λ^* and the one of λ^* on π are generally ambiguous. However, numerical experiments have shown that π is strictly increasing in q, at least for a linear search cost function and an arbitrary choice of the parameter p/k_0.

Conversely, when fewer firms enter, expected profit is

$$\pi(q) = \left[1 - e^{-\Phi(z)}\right] e^{-\lambda_A z} p \ , \ q > \bar{q} \ . \tag{7}$$

10

Again the first expression is the probability to receive at least one application (since the number of applications at every firm is Poisson distributed with mean $\alpha \lambda_A q = \Phi(z)$), and the second term is the probability that an active searcher gets no second offer, conditional on having one (since the number of job offers for active searchers is Poisson distributed with mean $\lambda_A z$). Importantly, expected profit in the range $q > \bar{q}$ does *not depend on the worker–firm ratio* q, since z and λ_A are independent of q. In contrast to standard search models, more entry does no reduce the chance to find a worker since the number of active searchers increases proportionately with the number of jobs. For the same reason, the chance that a contacted worker has another other offer does not increase with the number of job openings.

The equilibrium worker–job ratio balances marginal entry cost to expected profit:

$$\pi(q) = c(1/q) . \tag{8}$$

Whenever c is strictly increasing with appropriate boundary conditions, there is a unique solution to this equation. To summarize, an equilibrium is a worker–firm ratio q^* solving equation (8) together with the following search behavior of workers:

1. If $q^* \leq \bar{q}$, all workers search with common intensity λ^* which is the larger solution to equation (4).

2. If $q^* > \bar{q}$, share α of workers search actively with intensity λ_A, while all others remain inactive.

2.5 Response to shocks

Suppose that job surplus p increases (for example, because productivity goes up or unemployment income falls). For a *given number of firms*, such a shock has the following effects on search behavior. In a pure–strategy equilibrium, the common search intensity λ^* increases unambiguously in p. This follows immediately from the second–order condition which implies that marginal search costs are steeper than marginal return of search in equation (4) at the equilibrium λ^*.[8] In a mixed–strategy equilibrium, with linear search costs, both the number of active workers α and their

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[8]In this respect, the model differs from the one of Shimer (2004) where search intensity can also fall with higher job surplus.

search intensity λ_A are increasing in p (see equations (5)). Also the threshold value $\bar{q}$ increases; thus it becomes more likely that all workers are active. Intuitively, a larger job surplus raises the return to search, which unambiguously increases *search activity* and *search intensity* in this model for given q.

What is the effect of the positive shock on job creation? The impact on firm profit in the range $q > \bar{q}$ is unambiguously positive: a larger p raises the chance to find a worker (because more workers become active) and raises output in a filled job. In the range $q \leq \bar{q}$ the effect is less clear–cut. Although the chance to find a worker and job surplus go up again, the effect on the middle term in (6) is negative: the higher search intensity implies that workers are more likely to get a second offer in which case job profit would drop to zero. However, all my numerical experiments confirm that the overall impact of p on firm profit is positive. Hence, an increase in productivity raises the job–to–worker ratio $1/q$.

Figure 1 shows how job creation responds to such an increase in p, both in a pure–strategy equilibrium where all workers are active and in a mixed–strategy equilibrium where some workers are inactive, for different levels of the entry cost function. In the regime without inactive workers ($q \leq \bar{q} \approx .55$), the effect of a 20 percent increase of p on q is relatively modest: q cannot fall by more than 10 percent, even when entry costs are perfectly inelastic. In the regime with inactive workers, however, the reaction of q to an increase in p can be large: since $\pi(q)$ is flat in the range $q > \bar{q}$, the elasticity of q with respect to p tends to infinity when the elasticity of c with respect to $1/q$ tends to zero.

2.6 Efficiency

In the model of Albrecht et al. (2006), the decentralized equilibrium is inefficient along two margins: entry is excessive and workers send too many applications.[9] The first inefficiency also occurs in this model, but the second one must be qualified. In addition, another inefficiency emerges: it is never socially optimal that a fraction of workers remains inactive. To obtain these results, consider the problem of a social planner whose objective is to maximize total surplus per worker net of the costs of

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[9]Albrecht et al. provide a mathematical proof for the first inefficiency, but refer to numerical experiments for the second one.
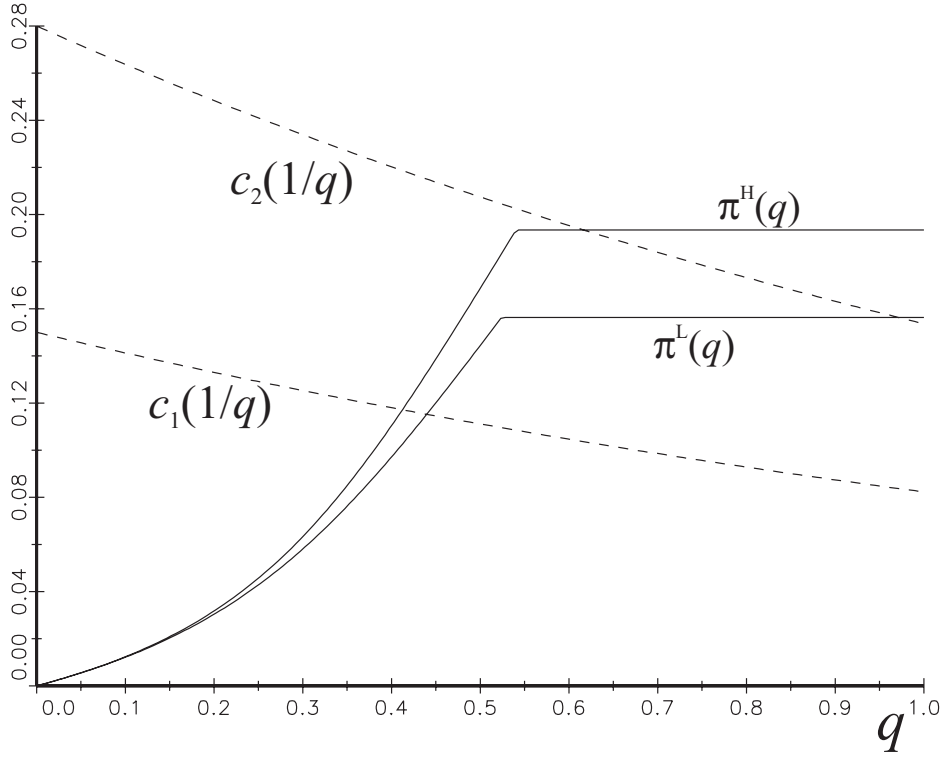
Figure 1: The response of the worker–firm ratio q to a productivity increase from $p^L = 1$ to $p^H = 1.2$ with $k(\lambda) = .1 \cdot \lambda$ for two exemplary entry cost functions c_1 (pure–strategy equilibrium) and c_2 (mixed–strategy equilibrium).

search and entry, with respect to λ and q:

$$\max_{\lambda, q} m(q, \lambda)p - k(\lambda) - \int_0^{1/q} c(v)dv .$$

Observe first that the planner's objective is strictly concave in search intensity λ. Thus, it is never optimal to let fraction α of workers search with positive intensity λ_A while others are inactive. The planner rather prefers that all workers search with the same common intensity $\alpha\lambda_A$. Generally, the planner's objective depends on a *distribution* of search intensities, rather than a common intensity as it is written here. However, it is easy to show that nonparticipation of a fraction of workers is

13

not optimal.[10] The social optimum satisfies the following first–order conditions:

$$k'(\lambda) = e^{-\frac{1-e^{-\lambda q}}{q}} e^{-\lambda q} p , \qquad (9)$$

$$c(1/q) = e^{-\frac{1-e^{-\lambda q}}{q}} \left[1 - (1+\lambda q)e^{-\lambda q}\right] p . \qquad (10)$$

Consider first the entry margin for a *given common search intensity* λ of workers. In the decentralized equilibrium, the worker–firm ratio equates profit as in (6) to marginal entry cost $c(1/q)$. On the other hand, the right–hand–side in the optimality condition (10) is strictly increasing in q and *smaller* than firm profit π. Hence, the worker–job ratio q is too small in the decentralized equilibrium; entry is excessive. The explanation for the inefficiency is a similar "business–stealing effect" as it is discussed in Albrecht et al. (2006, p. 877): the social benefit of an additional vacancy falls short of the private benefit since the vacancy can attract workers from other firms whose vacancies are then left unfilled. Put differently, wages do not internalize the negative externality that an entrant exerts on incumbents who might lose all their applicants to the entrant firm. The inefficiency could go away if the wage for workers with one offer was greater than the monopsony wage (for example, if there is Nash bargaining with an appropriate choice of worker bargaining power). Obviously, in the limit $\lambda \to \infty$ this business–stealing effect disappears and entry becomes efficient, reconfirming the results of Julien et al. (2000).

Consider now the search intensity margin for a given worker–job ratio $q \leq \bar{q}$. Search intensity in the decentralized equilibrium is the larger solution to equation (4). The right–hand–side of the optimality condition (9) is declining in λ. Hence, equilibrium search intensity λ^* is too large if and only if the right–hand–side of (4) is smaller than the right–hand–side of (9) at λ^* which is the same as

$$\frac{(1 - e^{-\lambda^* q})^2 e^{\lambda^* q}}{\lambda^* q^2} > 1 .$$

This inequality is true for all values of $q > 0$ and $\lambda^* \geq 1$, but it may be violated when $\lambda^* < 1$ and q is not too large. When all workers send more than one application on average, search is socially excessive since workers impose a negative externality on other workers: because firms cannot coordinate their job offers, some workers receive

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[10]Proof: in the mixed–strategy equilibrium, net surplus is $\alpha(m(\lambda_A, \alpha q)p - k(\lambda_A))$, but if all workers search with intensity $\alpha \lambda_A$, surplus is $m(\alpha \lambda_A, q)p - k(\alpha \lambda_A)$. Weak convexity of k implies $-\alpha k(\lambda_A) \leq -k(\alpha \lambda_A)$, and concavity of $1 - e^{-x}$ implies that $\alpha m(\lambda_A, \alpha q) < m(\alpha \lambda_A, q)$.

no offers while others receive multiple offers. This externality is not internalized; on the contrary, workers desire to receive two or more offers, so the incentive to send a large number of applications is strong. Indeed, with linear search cost function, λ^* is always larger than 1.[11] Hence in this case search is always excessive, but with other examples search intensity can also be too low. For example, with $k(\lambda) = [1 - (1 - \lambda/2)^{-4}]/40$, $q = 1$ and $p = 50$, the equilibrium at $\lambda^* = .918$ is smaller than the social optimum at $\lambda = .92$. Also in the calibration exercise of Section 3, search intensity turns out to be socially too low.

Proposition 3: *A mixed–strategy equilibrium where fraction $\alpha \in (0,1)$ of workers searches actively with intensity λ_A is never socially optimal: welfare would increase if all workers searched with common intensity $\alpha \lambda_A$. A pure–strategy equilibrium (q^*, λ^*) is not socially optimal since welfare can be raised by higher q at given λ^*. Moreover when $\lambda^* \geq 1$, welfare can be raised by lower λ at given q^*.*

3 Quantitative analysis

3.1 The dynamic model

I now embed the static model in a dynamic framework in discrete time t. The environment is stationary, so I consider a steady state equilibrium and confine the quantitative analysis below to comparative statics experiments. Existing jobs end with exogenous probability δ per period. There is no search on the job, so q is the number of unemployed workers per vacancy. Flow output in a filled job is p and flow unemployment income is b. The latter term is fully interpreted as unemployment benefits which are financed by (unmodeled) lump–sum taxes. Income from home production or within–family transfers are ignored. Workers want to maximize their expected utility

$$E \sum_{t=0}^{\infty} \beta^t [w_t + v(\ell_t)] \ ,$$

where β is the discount factor, w_t is wage or unemployment income in period t, and $v(\ell) = k \cdot (\ell^{1-a} - 1)/(1 - a)$ is flow utility of leisure, with $1 \neq a \geq 0$ and $k > 0$.

---

[11]This follows from (3) and the second–order condition which implies that $\lambda^* \geq \text{argmax}_\lambda[\lambda z^2 e^{-\lambda z}] = 1/z > 1$.

Thus workers do not care about fluctuations in labor income, either because they are risk neutral with respect to consumption, or because markets are complete. This assumption is a convenient simplification in most of the search–theory literature. Workers dislike variations in leisure, however, and parameter a measures the inverse of the elasticity of intertemporal substitution of leisure. All jobs are full time, and hours do not depend on productivity, so an employed worker consumes the fixed amount of $\ell_e \in (0,1)$ units of leisure. Leisure of an unemployed worker who searches with intensity λ is $\ell_u = 1 - \lambda/A$. Hence total time endowment per period is normalized to one, and A is maximum search intensity per period. Within every period, unemployed workers and vacant jobs are matched according to the same process as in the static model. Unmatched agents continue search in the next period.

In the following, I restrict attention to a pure–strategy equilibrium where all non-employed workers are active searchers (so they are classified as unemployed in the usual definition). It is straightforward to characterize mixed–strategy equilibria with inactive workers, but they do not deliver reasonable predictions in the numerical experiments. Specifically, in comparison with U.S. data, inactivity (i.e. nonemployed persons who want to work but do not search) becomes too volatile relative to unemployment, and vacancies vary too little relative to nonemployment (unemployment plus inactivity). Also the predictions of Proposition 2(b) are at odds with the evidence. Hence the inactivity mechanism of this model can only be quantitatively relevant when it is coupled with substantial heterogeneity.

As in the static model, there are jobs with high and low wages, depending on the number of offers a worker holds in his hand when leaving unemployment. Although low–wage earners may have an incentive to search for better–paid jobs, I assume that employers can identify the current employment status of an applicant and that employed workers can renegotiate when they obtain better offers later on. Under these assumptions, employers will never offer their vacant job to an employed worker, and search on the job does not take place.[12] Let V and J^j be the values of vacant and filled jobs with wages w^j in job status $j = l, h$. Suppose that flow costs of a vacant job are c, independent of the number of active firms. The Bellman equations

---

[12]This assumption is similar as in Julien, Kennes, and King (2006). In their model, however, low–wage earners search on the job, but only because of inter–firm productivity differentials, which are absent here.

are

$$
\begin{aligned}
V &= -c + \beta V + (1 - e^{-\lambda q})e^{-\lambda z}\beta(J^l - V) , \\
J^l &= p - w^l + \beta(1 - \delta)J^l , \\
J^h &= p - w^h + \beta(1 - \delta)J^h .
\end{aligned}
$$

In the first equation, firms gain only when they meet an applicant with no other offer, which happens with probability $(1 - e^{-\lambda q})e^{-\lambda z}$. Otherwise they either get no application or they offer the job to a worker who has at least one other offer. In this case, Bertrand competition drives the wage w^h so high that the value of the filled job is equal to the value of a vacant job, which is zero, so it follows that $w^h = p$. The free–entry condition $V = 0$ is

$$
c = (1 - e^{-\lambda q})e^{-\lambda z}\frac{\beta(p - w^l)}{1 - \beta(1 - \delta)} . \tag{11}
$$

For workers, let U and E^j be the utility value when the current state is unemployment or employment in a job of status $j = l, h$. Workers' Bellman equations are

$$
\begin{aligned}
U &= b + \beta U + \max_{\lambda \geq 0}\left[\varphi(\lambda z)\beta(E^h - U) + v(1 - \lambda/A)\right] , \\
E^l &= w^l + v(\ell_e) + \beta E^l + \beta\delta(U - E^l) , \\
E^h &= p + v(\ell_e) + \beta E^h + \beta\delta(U - E^h) .
\end{aligned}
$$

Here, $\varphi(x) = 1 - (1 + x)e^{-x}$ is the probability to find a high–wage job (that is, to obtain two or more job offers) for a worker who receives $x = \lambda z$ offers in expectation. Since workers with only one offer do not gain at the transition from unemployment to employment, $E^l = U$ must hold, which gives rise to the reservation wage equation

$$
w^l = b + v(\ell_u^*) - v(\ell_e) + \beta\varphi^*(E^h - U) , \tag{12}
$$

where $\ell_u^* = 1 - \lambda^*/A$ is optimal leisure of an unemployed worker and $\varphi^* = \varphi(\lambda^* z^*)$ is the equilibrium transition probability into high–wage jobs. The utility increase at such a transition is

$$
E^h - U = \frac{p - b - v(\ell_u^*) + v(\ell_e)}{1 - \beta(1 - \delta - \varphi^*)} . \tag{13}
$$

As in the static model, optimal search intensity equates marginal search costs to the marginal return from search:

$$
\frac{k}{A}\left(1 - \frac{\lambda}{A}\right)^{-a} = \lambda z^2 e^{-\lambda z}\beta(E^h - U) . \tag{14}
$$

17

A steady–state equilibrium is a vector $(z^*, \lambda^*, w^{l*}, (E^h - U)^*, q^*)$ solving the five equations (1), (11), (12), (13), and (14).

3.2 Calibration

There are nine parameters $(\beta, \delta, a, A, k, c, \ell_e, b, p)$ to choose together with a reasonable period length. Parameters are calibrated to match averages for the U.S. economy after December 2000, when the Bureau of Labor Statistics (BLS) began to measure job vacancies with the Job Openings and Labor Turnover Survey. Because the matching process requires that firms contact only one worker per period and are committed to the offer during the period, a period length of one or two weeks is appropriate. I deliberately specify a period to be a week long, as in the calibration of Julien, Kennes, and King (2006), though most quantitative results are similar with a two–week period. With an annual interest rate of 5%, this implies $\beta = .9991$. Shimer (2007b) estimates an average monthly separation rate (EU flow) of 1.73% for the period 01/2001–06/2007,[13] so I set $\delta = .0038$. The hazard rate of unemployment (job–finding probability) in my model is $h = 1 - e^{-\lambda z}$. With an average unemployment rate of 5.2% (between 12/2000 and 07/2007), this implies a mean hazard rate of $h = 6.93\%$ (which is also consistent with Shimer's estimation of the UE flow), and so $\lambda z = .0718$.

During the same period, the mean vacancy rate as reported by the BLS (i.e. vacancies divided by the sum of employment and vacancies) is $v = 2.43\%$. This yields a steady–state unemployment–to–vacancy ratio of $q = \delta(1-v)/(hv) = 2.202$. These numbers for q and λz together with equation (1) yield $\lambda = .0781$ and $z = .919$. Hence, my matching function calibrates U.S. data on unemployment and vacancies, provided that an average unemployed worker sends one application every twelve weeks and gets a job after after applying with a chance of 92%. The first number seems too small and the second one too large. However, the inclusion of an (irrelevant) mismatch parameter can help to circumvent this apparent oddness. Suppose that a worker applies at a given firm with probability λ_0/N, and that the firm observes match–specific productivity after it receives the application. With probability μ the worker fits the job, but with probability $1 - \mu$ the worker's productivity is so low that job surplus is negative and thus does not result in a match. Then

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[13]See http://robert.shimer.googlepages.com/flows

a given worker sends to a given firm an "effective application" (one that results in a match of positive surplus) with probability λ/N where $\lambda = \mu\lambda_0$. Therefore, in this interpretation, λ is the average number of *effective* applications per worker whilst λ_0 is the average number of *actual* applications. Similarly, λq is the average number of *effective* applications per vacancy. The actual number of applications and the probability to draw high match–specific productivity are irrelevant for the quantitative analysis. All that matters are the effective application rates. Likewise, parameter A is interpreted as the maximum average number of effective applications that a worker can send in a given period.

Regarding preferences, I set $a = 7.0$ to yield an elasticity of intertemporal substitution of leisure below the upper bound of .2 reported by Card (1994). Employed persons work 50% of their time and unemployed persons search 25% of their time. These numbers are arbitrary, though plausible choices. Given the above value of λ, $\ell_u^* = .75$ implies that $A = .313$. Productivity is normalized to unity, and unemployment income b is set at 29% of the mean wage, which is the replacement ratio of the U.S. in 1999 (see Table 2 of Nickell, Nunziata, and Ochel (2005)). The mean wage is $\overline{w} = \alpha w^l + 1 - \alpha$, where $\alpha = \lambda z e^{-\lambda z}/(1 - e^{-\lambda z}) = .965$ is the share of workers in low–wage jobs. Equations (12), (13) and (14) together with the requirement $b/\overline{w} = .29$ are then four equations in the unknowns b, w^l, $E^h - U$ and k which can be solved for the parameters $b = .26$ and $k = .0591$. The reservation wage is then $w^l = .891$ and the mean wage is $\overline{w} = .895$. Finally, (11) delivers $c = 3.396$. Note that vacancy flow costs are more than 10 times as large as in Shimer's (2005) calibration of the search and matching model. This is because wages are lower and job surplus is considerably larger here.[14] Alternatively, if wages were determined through Nash bargaining in those matches where a worker holds only one offer, job surplus (and so vacancy costs) would be much lower, but I decided not to follow that route.

Finally, it can be checked that utility from search is positive at these parameter values, so all unemployed are active searchers. Table 1 summarizes parameter choices and their explanation.

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[14]In Shimer's calibration, the wage is only 1.25 percent below productivity and the value of a filled job is as much as output of 1.7 weeks. In my calibration, the value of a filled job (which lasts more than 200 weeks on average) is the equivalent of 22 weeks' output.

Parameter	Value	Explanation
β	.9991	Annual interest rate 5%
δ	.0038	Monthly EU flow 1.7%
p	1	Normalization
b	.26	Replacement ratio 29%
a	7.0	IES of leisure below 0.2 (Card (1994))
ℓ_e	.5	Plausible leisure share of employed persons
A	.313	Plausible leisure share of unemployed persons ($\ell_u = .75$)
k	.0591	Unemployment rate 5.2% (via λ and $h = 1 - e^{-\lambda z}$)
c	3.396	Vacancy rate 2.43% (via q)

Table 1: Parameter choices.

3.3   The vacancy–unemployment relationship

How does the economy respond to a permanent, unanticipated change of productivity p or of the separation rate δ? Figure 2 shows BLS data on unemployment and vacancy rates for the period 12/2000–07/2007 in comparison to the model's steady–state response when p varies about 10 percent (between .97 and 1.07). The blue curve is the reaction of the baseline model which fits the data reasonably well, though the curvature of the model's Beveridge curve is too large. The red curve shows the model's reaction when search intensity is fixed; this would be the case in the limit of a zero intertemporal elasticity of substitution of leisure ($a \to \infty$). Generally, smaller values of this elasticity make the Beveridge curve steeper and larger values make it flatter.[15] Intuitively, at a low elasticity of substitution search intensity increases only little when productivity goes up, so unemployment falls only moderately relative to the increase in job creation. The reverse is true at larger values of the substitution elasticity. At the baseline calibration, despite low intertemporal elasticity of substitution, search intensity varies about 50 percent in response to a 10 percent productivity variation.[16]

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[15]With parameter a in the interval $[5, 9]$, the fit of the Beveridge remains reasonable, however.

[16]Shimer (2004) provides evidence against procyclical search intensity. He considers CPS data and uses two measures of search intensity. One is the "probability to search" which is irrelevant here since nonparticipation is not considered (all unemployed are searchers by definition). The other is the "number of job search methods". However, the variability of this measure is small
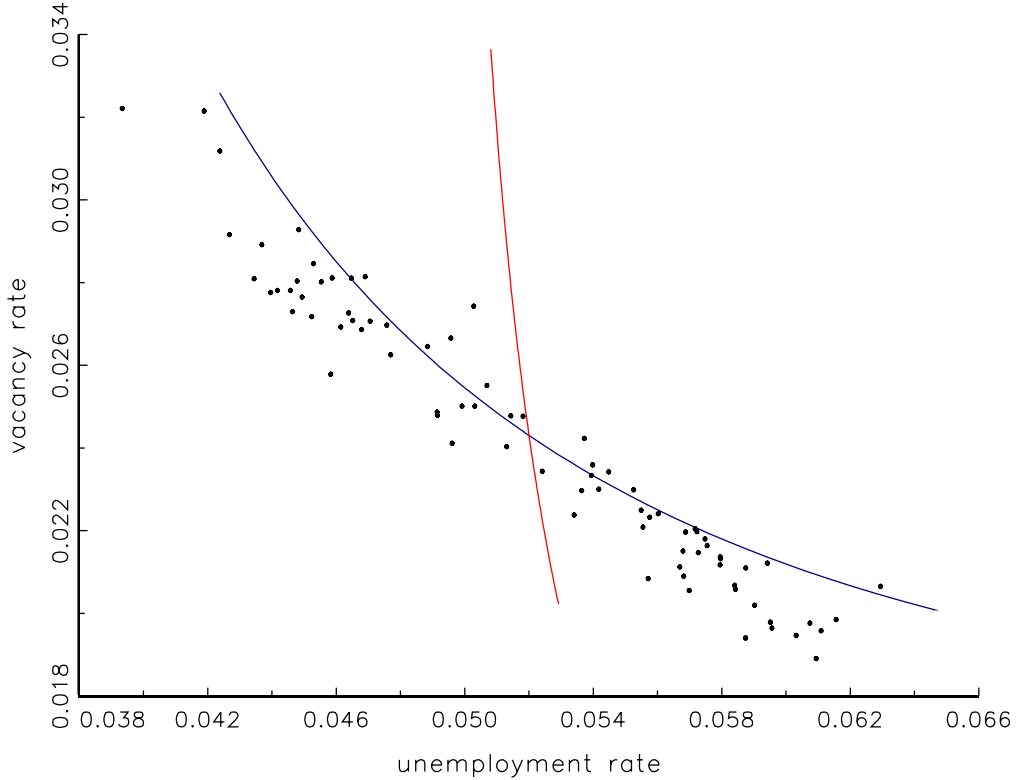
Figure 2: Beveridge curves generated by the model when productivity varies between .97 and 1.07 with variable search intensity (blue) and fixed search intensity (red). The dots show monthly data for the U.S. from 12/2000 to 7/2007.

On the other hand, if there is a permanent shock to the replacement rate, the vacancy rate and the unemployment rate move in the same direction. That is, the Beveridge curve shifts outwards (inwards) in response to increases (decreases) of the separation rate, which is also the case in the standard search and matching model. Shimer's data (see footnote 13) suggest however that the EU flow δ has moved only little during the last 6 years.

The baseline calibration with variable search intensity not only delivers the correct slope of the Beveridge curve, it also performs well along two other dimensions. One is the quantitative response of vacancies and unemployment to productivity

(Figure 7) and it is unclear how this number correlates with the actual search time and with the number of applications, which are relevant here.

shocks discussed in the next section. The other is the relationship between the vacancy–unemployment ratio $\theta = 1/q$ and the job–finding probability (hazard rate of unemployment) $h = 1 - e^{-\lambda z}$. Empirical estimates of matching functions suggest that the elasticity of h with respect to θ is in the range 0.3–0.5 (see Petrongolo and Pissarides (2001), p. 393). In my model, this elasticity depends on the source of the shock that drives variations in θ and h. However, a numerical calculation at the calibrated average values yields $d[\ln(h)]/d[\ln(\theta)] = .4765$ for changes in productivity and $d[\ln(h)]/d[\ln(\theta)] = .4772$ for changes of the separation rate; hence, these elasticities are about the same and fall into the range of plausible values. Figure 3 shows quarterly averages of monthly series for θ and h for the U.S. between 2001 and 2007, whilst the solid curve is the response of my model when productivity varies in the same 10 percent range as before. Again the fit of the model is reasonable when search intensity is variable, but not so much when it is fixed. In the latter case, the elasticity is only $d[\ln(h)]/d[\ln(\theta)] = .08$, clearly below the plausible range.

3.4 Response to productivity shocks

The model permits a quantitative response of the vacancy–unemployment ratio to a productivity shock which is of the same order of magnitude as in the data. This is already evident from Figure 3 which shows that a 10 percent variation in productivity induces a variation of the vacancy–unemployment ratio of about 100 percent. The local elasticity at the target value $p = 1$ is $d[\ln(1/q)]/d[\ln(p)] = 10.52$ which is more than six times as large as Shimer's (2005) elasticity of 1.71 in his calibration of the search and matching model. There are two explanations for this large difference. One explanation is variable search intensity which is procyclical and explains about half of the elasticity differential. In fact, with fixed search intensity the elasticity drops from 10.52 to 6.18 (see also the red curve in Figure 3). The other explanation is that flow utility of unemployment is much larger in the calibration of my model than in Shimer's calibration. More precisely, parameter b represents unemployment benefits, whilst the flow utility of unemployment (the flow opportunity cost of employment) is $\hat{b} = b + v(\ell_u) - v(\ell_e)$. At the parameter values of Table 1, unemployment income is at 29% of mean earnings, but flow utility of unemployment is much larger, at 83.5% of productivity and 93.2% of mean earnings. In Shimer's calibration, unemployment utility is arbitrarily set at 41 percent of the mean wage, and larger
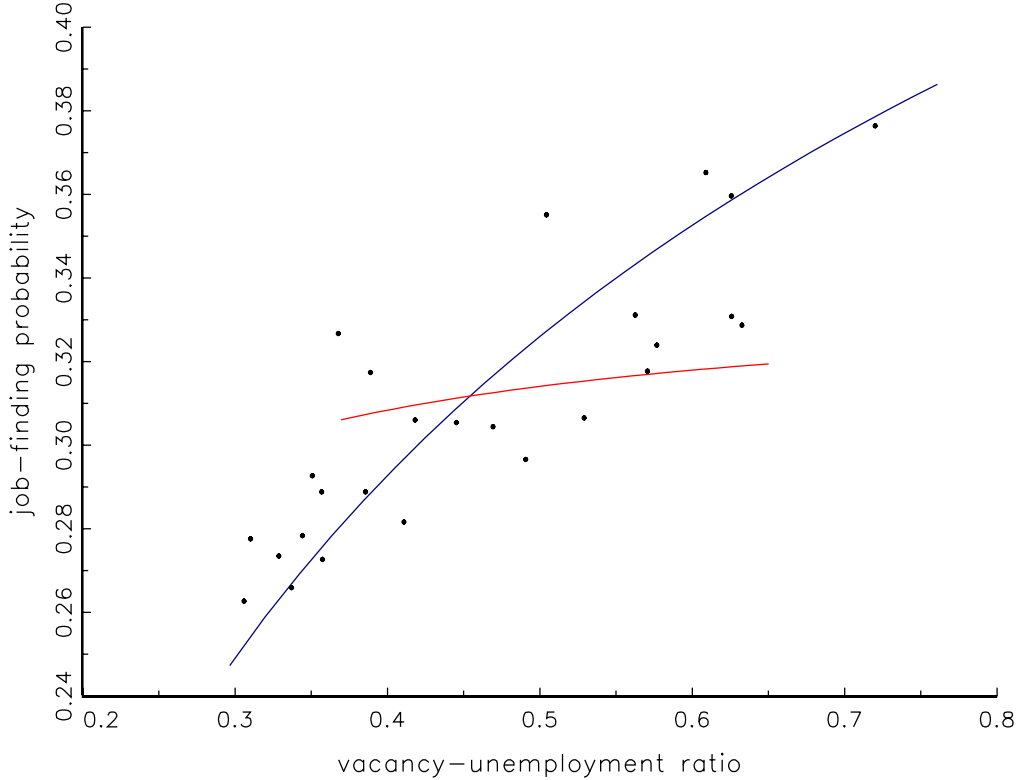
Figure 3: The relation between the vacancy–unemployment ratio and the job–finding probability when productivity varies between .97 and 1.07 and search intensity is variable (blue) and fixed (red). The dots show quarterly averages of monthly data for the U.S. from Q1/2001 to Q2/2007. The job–finding probability (UE flow) was constructed by Robert Shimer (see Shimer (2007b) and http://robert.shimer.googlepages.com/flows).

values of this number would magnify the economy's response to productivity shocks considerably.

In another approach to solve Shimer's (2005) puzzle on the amplification of shocks in search and matching models,[17] Hagedorn and Manovskii (2007) choose a value of unemployment utility which is even larger than mine (95.5% of productivity). They

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[17]There is already a substantial literature on this issue, a large portion of which argues in favor of some form of wage rigidity; see Hornstein, Krusell, and Violante (2005) and Yashiv (2007) for surveys.

pin down unemployment income and the worker bargaining power parameter (which is absent here) to match two targets: the wage elasticity with respect to productivity and a measure of vacancy costs. Hagedorn and Manovskii find that unemployment income must be large and that the worker bargaining power parameter is small which implies that worker surplus is small, too. My findings are similar in that unemployment income is large and that the mean wage is close to the reservation wage.[18] Indeed, about 96% of workers earn the reservation wage. But my calibration strategy is quite different from theirs: unemployment utility is decomposed into an income component and into a leisure component. The income component matches the replacement ratio for the U.S., whereas the leisure component follows from my choice of ℓ_e and ℓ_u, which are plausible, albeit arbitrary numbers. $\ell_e = .5$ is a common choice in the business–cycle literature. When I set $\ell_u = \ell_e = .5$, for example, the elasticity of the vacancy–unemployment ratio with respect to productivity falls to 1.37 (with variable search intensity) and to 1.17 (with fixed search intensity). However, leisure consumption should be considerably larger for unemployed than for employed persons. My choice of $\ell_u = .75$ seems low already, and larger values of this number would amplify the economy's reaction to productivity shocks even more.

3.5   Comparison with other models of coordination frictions

The bulk of the quantitative labor–search literature utilizes the approach of a black–box matching function (mostly Cobb–Douglas). On the other hand, the literature on microeconomic foundations of matching functions is by and large theoretical. Only recently, Shimer (2007a) and Mortensen (2007) have developed *mismatch models* which not only provide a novel structural foundation of the matching function, but which also deliver good quantitative predictions along the dimensions discussed in 3.3 above. However, models of unemployment which are based on *coordination frictions* (e.g. Montgomery (1991), Burdett, Shi, and Wright (2001), Julien, Kennes, and King (2000)) have not yet been analyzed with respect to their quantitative

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[18]Hornstein, Krusell, and Violante (2005) point out that a large value of unemployment income leads to an unrealistically large elasticity of unemployment with respect to unemployment income (see also Costain and Reiter (2003)). This is not true in my model: a 10 percent increase of b merely raises the unemployment rate by one percentage point (see Section 3.6 below).

implications.[19]   I will now demonstrate that the basic versions of these models, with homogenous agents, with a reasonable choice of the period length, *and without variable search intensity*, are unable to match the correct slope of the Beveridge curve and the right elasticity of the hazard rate with respect to the vacancy–unemployment relationship.

The model of Albrecht, Gautier, and Vroman (2006) is general enough to encompass the other coordination–failure models as special cases. In their model, every worker sends a applications, randomly to homogenous firms posting the same wage. After applications arrived, each firm makes a job offer to at most one worker, rejecting all others. The model gives rise to the hazard rate

$$h = 1 - \left[1 - z(aq)\right]^a , \tag{15}$$

where $z(aq) = (1 - e^{-aq})/(aq)$ is the probability to receive an offer conditional on applying and $q = M/N$ is the unemployment–vacancy ratio, with M and N large. The large–economy model of Montgomery (1991) and Burdett, Shi, and Wright (2001) is the special case $a = 1$, and the model of Julien, Kennes, and King (2000) is the limit $a \to \infty$. For given a ("fixed search intensity"), this matching function is not general enough to match both the unemployment rate and the vacancy rate, since q is the only free parameter. However, the introduction of match–specific productivity shocks enriches the model sufficiently to achieve this requirement (see also Petrongolo and Pissarides (2001)).[20] In Section 3.2 above, match–specific productivity shocks were introduced to interpret the model's calibrated parameters in a reasonable way. The actual (average) number of applications sent is irrelevant; what matters is the number of effective applications. In contrast, in the model of Albrecht et al. the number of actual applications is a structural parameter, and match–specific productivity is needed to match the model to the data. Again, assume that firms learn what applicants are suitable after they receive the applications. Let μ be the probability that a worker fits the job. With this modification, the probability to receive a job offer conditional on applying is $z_\mu(aq) = (1 - e^{-\mu aq})/(aq)$. Otherwise the matching function (15) stays the same.

---

[19]An exception is Julien, Kennes, and King (2000), but their focus is wage dispersion.

[20]Alternatively, one could try to use the number of applications a as a free parameter. However, to match mean unemployment and vacancy rates for the U.S. after Dec 2000 requires $a = .0197$, a number which is not compatible with the model which requires $a \in \mathbb{N}$.

I can now use the two parameters (μ, q) to target mean unemployment and vacancy rates. This requires again $q = 2.202$, while μ depends both on the choice of the period length and on the given number of applications. Given that workers send out applications only once (at the beginning of the period) and that firms contact only one applicant, a period length of one week is reasonable. With this choice I find that $\mu = .0752$ ($a = 1$), $\mu = .0383$ ($a = 2$), $\mu = .0078$ ($a = 10$), so the expected number of effective applications $a \cdot \mu$ is nearly constant and about the same as the value of λ in Section 3.2. Figure 4 shows the implied Beveridge curves induced by shifts in q, both for $a = 1$ and for $a = 100$ which are practically identical. With a period length of one week or two weeks, Beveridge curves are clearly too steep. A period length of one month would yield a good fit to the data, but such a choice is implausible for this model of coordination frictions; it would require that a firm can only contact one worker per month and that unemployed workers send out applications only once in every month. With a reasonably short period length, however, variable search intensity (as measured by parameter a in this model) would make the Beveridge curve flatter. Of course, this possibility is absent in the models of Montgomery (1991), Burdett, Shi, and Wright (2001) and Julien, Kennes, and King (2000).

With fixed a, also the hazard rate responds to little to variations in the vacancy–unemployment ratio. With a period of one week, I compute a local elasticity of the job–finding rate with respect to the vacancy–unemployment ratio of .0805 (for $a = 1$) and .0806 (for $a = 100$), clearly below range of plausible values. Again, variable search intensity can help to increase this number.

3.6   Welfare and policy

Results from section 2.6. show that equilibrium is not efficient in general: entry is excessive and search intensity can be too high or too low. The dynamic model requires a separate analysis of the welfare issue. One important difference is that there are unemployment benefits in the dynamic model which do not contribute to social welfare.[21] However, benefits induce a wedge between the private and the social job surplus, which dampens the incentives to search for workers and firms.

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[21]As mentioned before, benefits are financed by lump–sum taxes, so there is no net contribution to welfare. Of course, matters would be different if workers were risk averse and markets were incomplete.
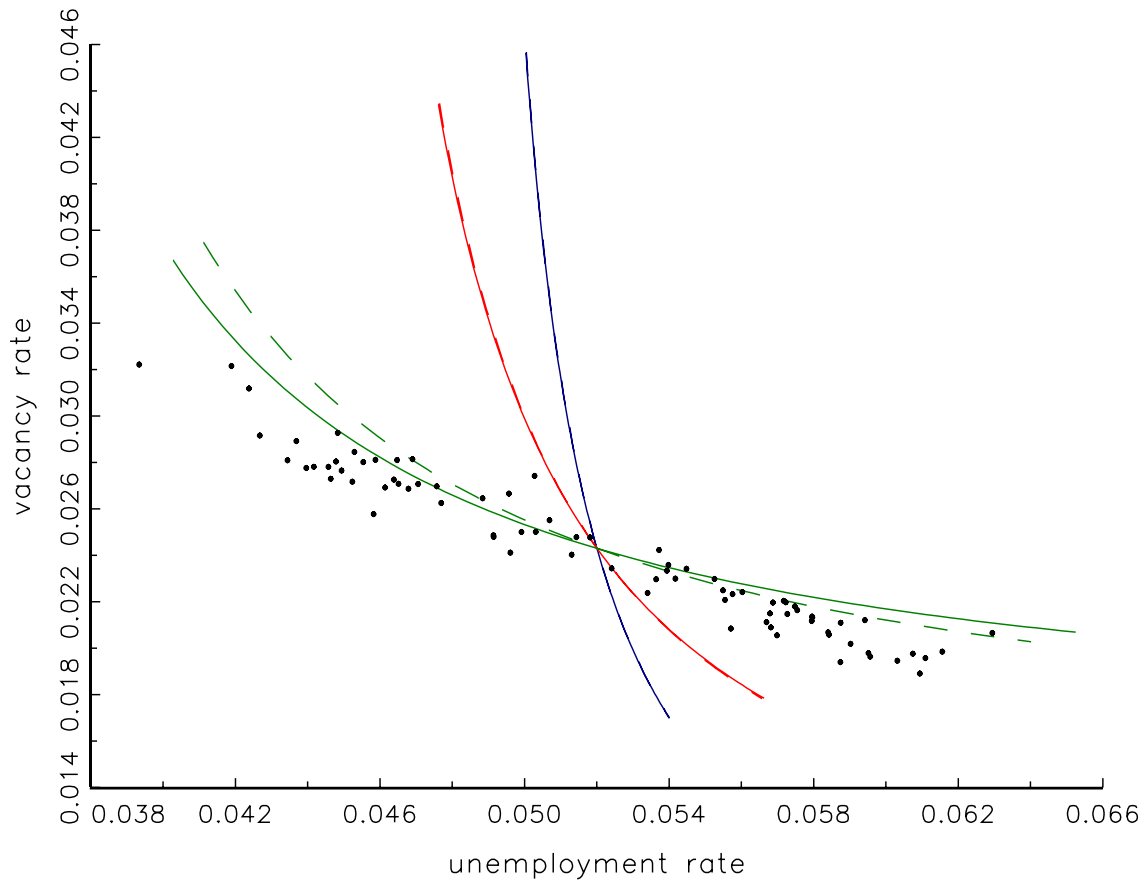
Figure 4: Beveridge curves generated by urn–ball matching models with $a = 1$ (solid) or $a = 100$ (dashed), when q varies from 1.101 to 3.303. The period length varies between one week (blue), two weeks (red) and one month (green). The dots show monthly data for the U.S. from 12/2000 to 7/2007.

This effect counteracts my previous findings that entry is excessive and that search intensity is excessive when $\lambda > 1$.

Because of quasilinear preferences, the planner's objective is to maximize a utilitarian welfare function which adds up the discounted value of the income stream net of entry cost and the discounted utility values of leisure. The recursive formulation of this problem is

$$W(u) = \max_{\lambda,q} \left\{ u \cdot v(1 - \lambda/A) + (1 - u) \cdot [p + v(\ell_e)] - \frac{cu}{q} + \beta W(u') \right.$$

$$\left. \text{s.t. } u' = u + \delta(1 - u) - m(q, \lambda)u \right\} . \tag{16}$$

Here u is the unemployment rate (the only state variable), $W(u)$ is welfare when current unemployment is u, and $m(q, \lambda)$ is the matching function (2). I obtain the following characterization of the social optimum.

Proposition 4: *In the social optimum, the unemployment–vacancy ratio q and search intensity λ are independent of the state variable u and satisfy the first–order conditions*

$$c = \beta e^{-\lambda z}[1 - (1 + \lambda q)e^{-\lambda q}]S , \tag{17}$$

$$\frac{k}{A}\left(1 - \frac{\lambda}{A}\right)^{-a} = \beta e^{-\lambda z}e^{-\lambda q}S , \tag{18}$$

where $z = (1 - e^{-\lambda q})/(\lambda q)$ and

$$S = \frac{p + v(\ell_e) - v(1 - \lambda/A)}{1 + \beta\delta - \beta e^{-\lambda z}(1 + \lambda z) + \beta\lambda e^{-\lambda z}e^{-\lambda q}}$$

is the social value of an employed person.

Proof: Appendix.

The intuition behind the optimality conditions is easy to explain. In (17), the term $e^{-\lambda z}[1 - (1 + \lambda q)e^{-\lambda q}]$ is the same as $d[m(q, \lambda)]/d[1/q]$, i.e. the number of new matches of an additional vacancy. Hence the term on the right–hand side is the social return of a vacancy which must be equal to marginal costs on the left–hand side. Similarly in (18), the term $e^{-\lambda z}e^{-\lambda q}$ is the same as $d[m(q, \lambda)]/d[\lambda]$, so the right–hand side is the marginal social return of an additional unit of search intensity.

When there are no unemployment benefits, it is straightforward to show that entry is excessive. Indeed, the free–entry condition (11), using (12) and (13), can be expressed as

$$c = \beta e^{-\lambda z}(1 - e^{-\lambda q})\frac{p - b + v(\ell_e) - v(\ell_u)}{1 + \beta\delta - \beta e^{-\lambda z}(1 + \lambda z)} . \tag{19}$$

It follows immediately that the right–hand side of (17) is smaller than the right–hand side of (19), at the same values of λ and q when $b = 0$. Since the right–hand

28

side of (17) is increasing in q, the equilibrium level of q is smaller than the socially optimal level of q (at given λ). Hence, for any given level of search intensity, there is too much entry. Obviously, this result can change when $b > 0$ is large enough.

Table 2 compares the equilibrium at the benchmark calibration with the social optimum. Search intensity is about 40 percent too low, and the vacancy–unemployment ratio $1/q$ is more than 400 percent too large. The planner would choose about the same level of the unemployment rate, but the desired vacancy rate would only be a quarter of what it is in the data.[22] Total welfare $W(u)$ (which is expressed in units the current output good) is about 20 percent below optimum. The last three columns of the table show the three components of flow surplus: total output, total utility from leisure, and vacancy costs per period. Output, leisure utility and vacancy costs are all smaller in the social optimum than in equilibrium.

	λ	q	$u(\%)$	$v(\%)$	Welfare	Flow output	Flow ut. of leisure	Vacancy costs
Equilibrium	.078	2.202	5.2	2.43	306.8	.948	-.591	.0802
Social opt.	.109	10.02	5.59	.59	368.24	.944	-.593	.0189

Table 2: Equilibrium versus social optimum.

There are different possibilities how policy can raise welfare in this economy. An increase in unemployment benefits reduces both search intensity and firm entry, so the net welfare effect is generally ambiguous. However, at the baseline parameterization, welfare rises with higher benefits. For example, a 10 percent increase in benefits yields $\lambda = 0.066$ and $q = 3.122$. The unemployment rate increases to 6.19% and the vacancy rate falls to 2.07%. Despite higher unemployment, welfare increases by 3.5 percent. Another policy with similarly ambiguous qualitative effects is a tax on profits. Quantitatively, however, the net effect of welfare is again positive. For example, a 10 percent tax on profits reduces search intensity to $\lambda = 0.0774$ and raises the unemployment–vacancy ratio to $q = 2.473$. The vacancy rate falls to

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[22]Note again that excessive entry is due to a business–stealing effect which emerges because wages do not internalize the negative entry externality on incumbent firms. Alternatively, if the low wage w^l was the outcome of Nash bargaining between worker and firm (instead of the monopsony wage), entry would be efficient for some value of the worker bargaining power parameter. Technically, the effect of Nash bargaining is the same as the one of a minimum wage discussed below.

2.2%, the unemployment rate increases to 5.3%; nevertheless welfare increases by 2.6 percent.

Lastly, consider the impact of a binding minimum wage. Formally the effect of a minimum wage is the same as an increase of worker bargaining power when wages in low–wage jobs are the outcome of a bargain between the firm and the worker with one offer. The minimum wage alters Bellman equations in that workers gain from a transition into low–wage jobs. Solving these equations yields

$$E^l - U = \frac{1}{1 - \beta(e^{-\lambda z} - \delta)} \left[\overline{w} - b - v(\ell_u) + v(\ell_e) - \beta\varphi^* \frac{p - \overline{w}}{1 - \beta(1 - \delta)} \right], \qquad (20)$$

which is positive when $\overline{w}$ is a binding minimum wage greater than w^l. Also the value of a high–wage job changes to

$$E^h - U = \frac{1}{1 - \beta(e^{-\lambda z} - \delta)} \left[\overline{p} - b - v(\ell_u) + v(\ell_e) - \beta\lambda z e^{-\lambda z} \frac{p - \overline{w}}{1 - \beta(1 - \delta)} \right]. \qquad (21)$$

Equilibrium search intensity changes from (14) to

$$\frac{k}{A} \left(1 - \frac{\lambda}{A} \right)^{-a} = \lambda z^2 e^{-\lambda z} \beta(E^h - U) + z e^{-\lambda z}(1 - \lambda z)\beta(E^l - U). \qquad (22)$$

A steady–state equilibrium with a binding minimum wage $\overline{w}$ is a vector $(z^*, \lambda^*, (E^h - U)^*, (E^l - U)^*, q^*)$ solving the four equations (1), (20), (21), and (22) and the free–entry condition (11) with $\overline{w}$ replacing w^l.

Qualitatively, the minimum wage raises the search return for workers and lowers the one for firms, so it takes a positive impact on λ and q, enhancing welfare unambiguously. In the numerical example, even a minimum wage which is one percent above the reservation wage of the laissez–faire economy has a substantial welfare effect. With $\overline{w} = 1.01 \cdot w^l = .9$, I find that search intensity increases to $\lambda = .0792$ and that q increases to 2.388. The vacancy rate falls to 2.23% and the unemployment rate falls slightly to 5.18%, despite reduced entry. Total welfare increases by 2.3%.

4 Conclusion

This paper develops a model of coordination frictions with variable search intensity in which equilibrium can be fully characterized by a simple set of first–order conditions. Under bad labor market conditions, however, some workers cease to search.

This phenomenon emerges without any heterogeneity in the workers' productivities or in their tastes for leisure. Such a "discouraged–worker effect" is not socially desirable; the planner desires that all homogenous workers search with a common intensity. But even under better labor market conditions, equilibrium is typically inefficient, as there are too many vacancies and a suboptimal level of search intensity. In the quantitative part of this paper, I argue that variable search intensity improves the fit of coordination–friction economies. It makes the Beveridge curve flatter and raises the elasticity of the matching function with respect to the vacancy–unemployment ratio. It also helps to increase the economy's quantitative response to productivity shocks.

The following two extensions are left for future work. The first is the effect of heterogeneity on search intensities. On the one hand, when there are jobs with high and low productivities, it is interesting to know whether workers search too hard for the good jobs and too little for the bad ones, and how much they diversify their "search portfolio". A similar issue is analyzed in Gautier and Wolters (2007). On the other hand, when workers are endowed with different skills, it should be understood how much search intensities between these groups differ, and what the implications are for skill differences in wages and employment rates.

The second extension concerns the mechanism of wage determination. In many search models, firms post wages (and are committed to these announcements) before workers send applications. In one strand of the literature, known under the label "competitive search", workers observe all wages and direct their search to specific wages. Equilibrium under competitive search is typically constrained efficient. In urn–ball matching models, such results have been obtained when workers send one application (Burdett, Shi, and Wright (2001)) or several applications (Kircher (2007)). Kircher also proves that search intensity, as measured by the number of applications, is constrained efficient. In my model, unlike competitive search, workers do not observe all job openings; instead they decide how much time they spend on sampling jobs and they apply at all jobs they find suitable. Thus search at the application stage cannot be directed towards specific wages, though it may be directed towards job types, as discussed in the previous paragraph. Hence, a variation of this model with posted wage would rather resemble the random–search model of Burdett and Judd (1983) than competitive search.

Appendix

Proof of Proposition 1:

Consider some worker i and suppose that all other workers search with the same intensity λ, so that the probability to get an offer conditional on applying is $z(\lambda) = (1 - e^{-\lambda q})/(\lambda q)$. The best–response of worker i is either at $\lambda_i = 0$ which gives rise to utility $U_z(0) = 0$, or at the larger positive solution of (3) with $z = z(\lambda)$, denoted $\lambda_i = R(\lambda)$. This positive solution exists if, and only if,

$$k_0 \leq \max_{\lambda'}[\lambda' z(\lambda)^2 e^{-\lambda' z(\lambda)} p] = z(\lambda) e^{-1} p \ ,$$

which can also be expressed as $\lambda \leq \Phi(k_0 e/p)/q$. Since $z(\lambda) \leq 1$, this requires in particular that $p/k_0 \geq e$. If worker i searches with intensity $R(\lambda) > 0$, substitution of (3) into the objective function gives

$$U_{z(\lambda)}[R(\lambda)] = p\Big\{1 - \big[1 + R(\lambda)z(\lambda) + (R(\lambda)z(\lambda))^2\big]e^{-R(\lambda)z(\lambda)}\Big\} \ ,$$

which is non–negative iff $R(\lambda)z(\lambda) \geq x \approx 1.79328$. Using (3) again, the inequality

$$z(\lambda) = \frac{k_0 e^{R(\lambda)z(\lambda)}}{pR(\lambda)z(\lambda)} \geq \frac{k_0 e^x}{px}$$

is equivalent to $R(\lambda)z(\lambda) \geq x$ (so it is equivalent to $U_{z(\lambda)}(R(\lambda)) \geq 0$) since e^y/y is increasing in $y > 1$ and since $R(\lambda)z(\lambda) > 1$ holds. Using the definition of $z(\lambda)$ and Φ, this inequality can also be expressed as

$$\lambda \leq \frac{1}{q}\Phi\Big(\frac{k_0 e^x}{px}\Big) = \overline{\lambda} \ .$$

Hence, for any $\lambda \in [0, \overline{\lambda}]$, the best response of worker i is at $\lambda_i = R(\lambda) > 0$. Moreover, $\overline{\lambda} > 0$ iff $p/k_0 > e^x/x$. An equilibrium in pure–strategies amounts to finding a fixed point of $\lambda^* = R(\lambda^*)$ satisfying $0 \leq \lambda^* \leq \overline{\lambda}$. Since R is continuous and satisfies $R(0) > 0$ (from $p/k_0 > e$), such a fixed point exists if $R(\overline{\lambda}) \leq \overline{\lambda}$. But by construction,

$$R(\overline{\lambda}) = \frac{x}{z(\overline{\lambda})} = \frac{px^2}{k_0 e^x} \ .$$

Hence, $R(\overline{\lambda}) \leq \overline{\lambda}$ is equivalent to $q \leq \overline{q}$. $\qquad\qquad\square$

Proof of Proposition 4:

As in section 7.2 of Rogerson, Shimer, and Wright (2005) it is straigtforward to show that $W(.)$ is affine–linear; it takes the form $W(u) = w_0 - S \cdot u$ where S is marginal social value of an employed worker. Differentiation of (16) with respect to u together with the envelope theorem yields

$$S = p + v(\ell_e) - v(\ell_u) + c/q + \beta S(e^{-\lambda z} - \delta) \, , \tag{23}$$

with $\ell_u = 1 - \lambda/A$. The first–order conditions for λ and q are

$$v'(\ell_u)\frac{1}{A} = \beta e^{-\lambda z} e^{-\lambda q} S \, , \tag{24}$$

$$c = \beta e^{-\lambda z}[1 - (1 + \lambda q)e^{-\lambda q}]S \, . \tag{25}$$

These conditions confirm that optimal λ and q do not depend on the state variable u, so $W(.)$ is indeed affine–linear. Both first–order conditions are the same as in Proposition 4. The value S follows after substitution of (25) into (23) and solving for S. $\qquad\square$

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