Should Public Retirement Plans be Fully Funded?

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Abstract

Most state and local pension plans strive for full funding, at least by actuarial standards. Funding measured at market values fluctuates and often falls short. A common argument for full funding is that pensions are a form of deferred compensation that does not justify a debt. The paper examines public finance, political economy, and financial market issues that bear on optimal funding, broadly and in a series of models. I find full funding difficult to justify except under extreme assumptions. Importantly, most taxpayers hold debt and face borrowing costs, and this makes pension funding costly.

"Neither a borrower nor a lender be"

(Shakespeare)

1. Introduction

Most state and local retirement plans strive for full funding as measured by actuarial standards. Funds are commonly invested in risky assets. Hence actual funding ratios—the ratios of assets to accrued benefit obligations—fluctuate and are often less than 100 percent. Press reports about underfunding naturally cause taxpayer anxiety. Recent experience follows this pattern: though average funding ratios were over 80% before the recent financial crisis, many state and local pension funds are now seriously underfunded (Munnell et al. 2008, 2010). Concerns about funding are reinforced by controversies about actuarial standards. A key dispute is about discount rates. Economists have argued that officially reported funding ratios are inflated because actuarial rules prescribe discount rates on pension obligations that are too high (Novy-Marx and Rauh 2009).

These concerns raise questions about pension funding. What is full funding? Is it optimal? Should it be required at all times or only in expectation? How should government obligations be reported to the public? These questions have important ramifications for financial markets and for state and local governments. Full funding all the time would require overfunding on average and perhaps restrictions on investments. Giving politicians discretion would undermine balanced budget restrictions and thus alter the political economy of state and local governments.

Questions of optimal funding are complex because they are intrinsically linked to a range of challenging economic, financial, and political economy issues. This includes federal tax rules as motivation for deferred compensation; the risk sharing implications of defined benefit plans (DB) as compared to defined contributions (DC); the political economy of balanced budget rules; the equity premium and its ramifications for investment strategies; and labor issues relating to career employment.

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This paper will first review these issues broadly and then examine them a series of models. The main conclusion is that optimal funding depends on taxpayers' cost of funds and on the presence of legal ambiguities and uninsurable default risks. Taxation to accumulate pension assets is inefficient whenever the risk-adjusted return to assets is less than taxpayers' marginal cost of funds. Taxpayers who hold debt and face intermediation cost will favor zero pension funding. This is relevant because over 75% of U.S. families hold debt. Funding is optimal only if collateral is needed to mitigate legal risks or default risks. Full funding is not optimal except under extreme assumptions.

The analysis has implications for pension accounting and for the tradeoff between DB and DC plans. First, the appropriate discount rate for unfunded DB pensions is the marginal cost of funds of taxpayers. If most taxpayers are borrowers, appropriate discount rate is a risk-adjusted *borrowing* rate—a higher rate than the safe interest rate. Second, because funding is costly and full funding is usually suboptimal, regulations that impose full funding would undermine employer incentives to offer DB plans. Most private employers phased out their DB plans after costly funding and insurance requirements were imposed in the 1970s. If current anxieties about public retirement plans lead to excessive funding requirements, the effects may prove similarly destructive.

The paper is organized as follows. Section 2 reviews pension funding in general, comments on the relevant literature, and on funding policies, actuarial standards, and investment strategies of public pension funds. Section 3 sets up a stochastic overlapping generations model with local property taxes to study pension funding. Section 4 examines the impact of legal ambiguities and default risk; Section 5 presents a model without property taxes. Section 6 comments on incomplete financial markets. Section 7 concludes.

2. The Problem of Pension Funding

This section examines several conceptual issues that make pension funding problematic.

A key distinction is between Defined Benefit (DB) and Defined Contribution (DC) plans. In DC plans, the employer promise is limited to making contributions. In DB plans, in contrast, funding is choice, and unfunded promises create obligations for the employer just like a debt. Pension obligations are more complicated than debt because they are often annuitized, contingent on earnings records, and inflation indexed. There are equally safe, however, due to strong legal protections against default (see Brown and Wilcox 2009; Peng 2009). Because DC plans are funded by construction, funding questions are about DB. About 80% of state and local pension plans are DB plans (Munnell et al 2008).

2.1. The Ambiguous Meaning of Full Funding

There are at least three conceptually different ways to interpret full funding. I will call them the accounting view, the economics view, and the popular view.

The accounting view considers a pension fully funded if an actuarial measure of fund assets at some date equals an actuarial measure of accrued liabilities. Accounting rules for U.S. state and local governments are set by the Government Accounting Standards Board (GASB). The key rule for pension accounting, GASB 25, gives plan sponsors a choice between several different actuarial methods to compute pension obligations (see Peng 2009). One measure, the accrued benefits obligation (ABO), seeks to determine the present value of benefits earned by current plan participants at the valuation date, assuming no accrual of obligations in the future. All others are projection methods, which involve (1) estimating the lifetime benefits of current participants, including projected future earnings, (2) allocating the cost to past and future service, and (3) treating the past-service component as accrued liability. The most commonly used projection method is Entry-Age Normal (EAN), which allocates cost in proportion to earnings (since entry).

Importantly, GASB gives plan sponsors wide discretion in discounting future benefits and allows them to tie the discount rate to the expected return on assets. On the asset side,

smoothing methods are common that spread the recognition of a capital gain or loss over several years. This means the actuarial value of assets is a moving average of market values. Measures of funding are the *unfunded actuarial accrued liability* (UAAL), the gap between actuarial liabilities and assets, and the *funding ratio*, which is the ratio of actuarial assets over liabilities. If a plan sponsor makes contributions under GASB rules, the plan should be fully funded on average, but the funding ratio will almost always differ from 100% due to capital gains and losses. GASB rules specify that any positive or negative UAAL should be amortized over time by raising or lowering new contributions relative to the "normal" contributions.

The economics view considers a pension fully funded if the market values of assets equals the present value of promised pensions (e.g., Novy-Marx and Rauh 2009). Distinctive are the rejection of smoothing methods to value assets, the use of state-contingent claims pricing to value future benefits, and the use of economic reasoning to ascertain the scope of obligations (Bulow 1982). Notably, expected asset returns are rejected as discount rates for liabilities. Similar to accounting view, underfunding is measured at a point in time, and degrees of funding are computed as ratios or differences of assets and liabilities.

By popular view, I mean the notion that a pension plan is fully funded if there is no risk of a funding shortfall that might require compensating contributions from the plan sponsor in the future. In the press and in the political debate, the central issue is that taxpayers might have to "bail out" public pension funds. The probability of a shortfall might be suitable as a quantitative measure of funding risk.

Zero risk of a shortfall is a much higher standard than the others, and perhaps impossible to meet. Full funding in the economic or accounting sense does not rule out underfunding in the future. Indeed, the odds should be about 50:50 with symmetric shocks and fair accounting. There are only two ways to guarantee no future shortfall: full funding combined with perfect hedging of all risks, or extreme over-collateralization that ensures full economic funding in all states of nature. Perfect hedging is difficult because pensions are

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claims contingent on complicated compound events. Collateral is costly, *per se* and because excess funds are difficult to recover by the sponsor.

Uncertainty about future funding ratios can be reduced significantly by a "safe" investment strategy; and then a modest over-collateralization may rule out future shortfalls with near certainty. Thus the popular view calls in effect for generous funding combined with a low risk investment strategy—or full funding in the economic sense plus a buffer for unavoidable fluctuations.

A practical way to ensure a buffer may be by insisting on fiscally conservative assumptions for the valuation. This may explain why allegations of underfunding receive so much public attention. If the public is concerned about risk, a study declaring a pension plan underfunded under pessimistic assumptions is troubling, even if the same plan is certified as fully funded by the accountants and perhaps overfunded in a variety of much more likely scenarios.

Finally, one should ask why funding matters. As noted by Lucas and Zeldes (2009) pension funding and investments would be irrelevant if Ricardian neutrality applied. D'Arcy et al. (1999) and Lucas-Zeldes argue that funding matters because taxes are distortionary, and hence optimal funding should be governed by tax smoothing arguments. D'Arcy et al. conclude that a range of funding levels can be optimal, depending on the growth rates of taxes and expenditures. Lucas-Zeldes find that optimal tax smoothing favors (at least some) equity investment. Bader and Gold (2007) argue that public pensions matter for reasons of tax arbitrage. They favor bond funding to help local taxpayers minimize federal taxes. Because discount rates are controversial, my analysis below will focus on credit market imperfections, starting from a neutral benchmark.

The multiple meanings of full funding expand the question in the title. The question is not only if pensions should be fully funded, but also in what sense and why it should matter.

2.2. Deferred Compensation and Balanced Budget Rules

Pension promises are a form of deferred employee compensation. In exchange for a future pension, employees accept a reduced current salary.

Almost all states and local governments operate under balanced budget rules. The operating budget (general fund) is supposed to be balanced every year. A separate capital budget allows for bond financing, but usually subject to voter approval. Bonds are repaid out of operating funds as the capital asset depreciates. Pension funds are organized as separate entities, funded by contributions from the operating budget and dedicated to paying retirees. If pensions are fully funded, this budget framework ensures that government net worth is positive.

Underfunded pension obligations undermine this fiscal framework. A shortfall due to unexpectedly low investment returns reduces government net worth. A shortfall because of missed contributions means that the operating budget understates employee compensation. Either way, a funding gap encumbers future operating funds just like a bonded debt. A balanced budget requirement is ineffective if the budget can be balancing by reducing current wage payments in exchange for unfunded pension promises. Thus pension funding is intrinsically linked to the political economy of balanced budget restrictions.

This raises two questions. First, what motivates DB pension plans in the public sector? Tight restrictions are more defensible if DB plans have no apparent rationale apart from evading balanced budget rules, but not if there are efficiency argument in favor. Second, what justifies balanced budget rules? Without such rules, a special purpose pension fund would be meaningless. Citizens might still be interested in measuring pension obligations to estimate future taxes, but funding would be interchangeable with other public funds.

2.3. What Motivates Employer Pensions?

The many arguments for employer pensions fall into three main groups—taxes, risk sharing, and human capital/labor issues.

The tax argument is straightforward. Compensation paid in form of a pension is taxable only when the pension is paid out. Hence deferred compensation means deferred taxes. Assets in funded plans compound without being taxed repeatedly. Not surprisingly, pensions—both private and public—became popular after WWII, at a time of high marginal taxes on regular, non-sheltered savings. Note, however, that the tax argument applies to all deferred compensation and does not provide a rationale for DB plans. A DC plan would provide the same tax shelter with less employer entanglement.

Risk sharing provides a clear distinction between DC and DB plans. By construction, the employer has no obligation to a DC plan beyond making the initial contribution. In a DB plan, in contrast, the employer is responsible for specific benefits.

There are two microeconomic risk-sharing issues that favor DB pensions. One is adverse selection in the market for life annuities. Annuities are less subject to adverse selection when provided to employees as group than if employees tried to buy annuities individually. Such risk pooling is automatic in a DB pension, but difficult to implement in a DC plan.[1] Second, DB investments can be more diversified as they may include "alternative" asset classes not easily held in a DC plan.[2] Both issues are microeconomic in the sense that risks is shared among employees, so the benefit has essentially no cost to the employer.

There are also two sets of "aggregate" risks, one tied to funding strategy, the other tied to the benefit design. The funding-related uncertainty is about real investment returns. In a DB plan, the sponsor in effect owns the risk and returns of the investment portfolio—though under some conditions, the risk is shared with working age plan members (see Bohn 2010). Benefit-related uncertainty is caused by a variety to features that make the real cost of pensions payments stochastic, e.g., an indexation of benefits to future wages, deviations from one-for-one indexing to inflation, and uncertainty about beneficiaries' collective longevity.

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[1] In DC plans, retiring members invariably demand a choice between annuities and a continuation of mutual fund investments, which makes the annuity option subject to adverse selection.

[2] A related issue is management expertise and cost. Before low-cost mutual funds and computerized bookkeeping became widespread, a single pooled DC account was clearly cheaper to manage than individual accounts. Even now, a significant fraction of DC participants seem to manage their investments inefficiently or lack access to low cost investment options.

Human capital issues are complex and arguably job and firm specific, but relevant. DB pensions are commonly tied to final salaries times years of service, which means they favor long-reserving employees who earn raises throughout their career. They penalize employees who quit early—with final nominal salary far below the expected end-of-career level—or are unsuccessful. Hence a DB pension can serve as an incentive or retention mechanism.

The specifics depend on the labor market. One polar case is a competitive labor market. Then total compensation at all times equals the marginal product of a worker's wage. Pension accruals that increase with age would be reflected in a flatter age-salary profile, and restrictions on mobility due to pensions would be inefficient. A second polar case is a unionized work place with seniority system, where young employees are paid less than their marginal product in exchange for an entitlement to earn a premium later. A DB pension can facilitate rewards for seniority. More generally, firm-specific human capital and frictions to mobility may make labor relationships imperfectly competitive. A DB pension can help facilitate implicit contracts that encourage efficient investment in human capital; but it might become an obstacle to job mobility.

It is an open question why the public sector has not followed the private sector trend towards DC plans. DB pensions were common in the private sector in the 1950s to 1980s. Since then, many private firms have terminated their DB plans and shifted to DC or Cash Balance (hybrid) pension schemes.

One line of explanation is that the demise of private sector DB pensions is inefficient and regrettable—a result of bad regulation and credit risk. A mandatory pension guarantee systems is in effect a tax on well-funded plans. ERISA gave creditworthy employers incentives for to convert to DC, causing adverse selection and triggering a death spiral of more plan terminations and adverse selection. In this view, DB plans in the public sector are still efficient because they are not subject to the same regulations. The demise of private DB plans may even be a comparative advantage for public employers.

An opposing view holds that the move from DB to DC in the private sector was an efficient response to increased job mobility, which suggests a reduced importance of firm-specific human capital. In this view, public sector's failure to follow is signal of inefficiency, perhaps due to inertia or pressure from public sector unions. Career employment remains prevalent in the public sector. One may argue that this is efficient, e.g., to avoid conflicts of interest ("revolving doors") or due to specialized training, say, for police officers; but the argument is not obvious and it may not apply to all public sector jobs.

While these human capital and labor relations issues should be noted, they remain relatively unsettled and are not the focus of this paper.

2.4. Balanced Budget Rules and Full Funding

Balanced budgets are extremely popular according to opinion polls, even at the federal level.[3] Full funding of pensions has a similar popular appeal to fiscal conservatism. A failure to make pension contributions is easily understood as violation of the balanced budget principle.

The popularity of balanced budgets is a challenge to economic theory. Theories of optimal dynamic taxation call for unbalanced budgets essentially all the time, because deficits and surpluses help to stabilize tax rates when there are fluctuations in spending and in the tax base. Hence a positive theory of balanced budget and other fiscal rules requires a second-best setting where political economy frictions are sufficiently severe to override tax smoothing objectives.

The political economy literature is somewhat unsatisfactory with regard to normative analysis at the state and local level. The theoretical literature has focused on national debt and identified a range of circumstances that favor excessive debt (see, e.g., Persson and Tabellini, 2000; Alesina and Perotti, 1995). A constitutional rule imposing balanced budgets is a natural corrective mechanism for such distortions. But because rigid rules are costly, theory suggests that rational voters should demand sophisticated balanced budget rules that are conditional or

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[3] For example, according to a Nov.2009 CNN poll, 67% of U.S. voters agree that "the government should balance the budget even when the country is in a recession and is at war," suggesting support for balanced budgets even under extreme conditions.

cyclically adjusted. The empirical literature has focused practical questions about the effects of different rules, taking their existence as given.

Political economy models of national debt are in principle applicable to state and local governments. Notably, government authority is often divided among multiple agents. Hence strategic interactions between them can create common pool problems, and responses to fiscal shocks may be delayed. Uncertainty about reelection is also likely to shorten politicians' planning horizons and invite the use of debt as tool to tie the hands of successor governments.

However, there may be simpler principal-agent explanations of why voters like balanced budgets at the local level. Voters must monitor politicians who act as their agents. Credible information about local budgets is often unavailable or costly, e.g., requiring time to attend meetings. Each voter must monitor are multiple entities—the city, the county, the state. If politicians have no authority incur debt, the potential damage from political favoritism, corruption, or other monitoring failures is bounded by current revenue, whereas damages could be huge if debt were allowed.[4] Simple rules also economize on information cost. A headline saying the balanced budget rule was violated is much easier to understand than a complicated budget document. The fact that voter approval makes bond issues acceptable is consistent with the monitoring story.

A full funding requirement for public pensions is appealing for similar reasons of simplicity and clarity. There are problems, however, because funding estimates require numerous assumptions that might be manipulated by politicians trying to evade balanced budget constraint. Moreover, a fund must be managed, and this creates new monitoring problems.

One problem is the choice between accrued benefits and projected benefits. In an ongoing DB plan with benefits linked to final salary, accrued benefits are usually smaller than

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[4] Such information problems do not imply "fiscal illusion"—the notion that voters underestimate the cost of deficit-financed expenditures because they observe the benefits but not the long-run cost. Persistent one-sided errors are difficult to reconcile with basic rationality and seem implausible in this age of cynicism about politicians. The issue here is risk reduction. An appropriate analogy is the question how much signature authority to give to an accountant—control over current accounts or also the power to borrow.

projected future benefits. If the employer has a right to terminate the plan, one would have to appeal to convoluted reputational arguments to justify a value greater than currently accrued benefits (Bulow 1982).[5]

A second problem area is discounting. The methods prescribed by accountants are economically unjustified, as noted above. Moreover, pensions are complex contingent claims, which means the correct discount rates are difficult to determine and controversial even among economic experts. Such ambiguities are especially troubling if reported funding gaps serve as summary statistic to information-constrained voters. A pension fund that is fully funded under some (reasonable) assumptions can easily be portrayed as underfunded by making more unfavorable "conservative" assumptions.

For instance, suppose a voter tries to assess if a politicians violated the balanced budget principle by making unfunded pension promises. A rational assessment must trade off type-I versus type-II errors. Fiscally conservative assumptions are appropriate if the damage from underfunding is greater than the cost of overfunding. While weak funding rules undermine the balanced budget rule, overly restrictive rules would discourage the establishment of pensions plans even when they are efficient.

Recent changes in accounting for retiree health benefits illustrate this tradeoff. GASB 43 treats the present value of projected future employer payments for retiree health as a liability even, if these payments are not vested—i.e., even if employee contributions could be raises or benefits canceled at any time. This recent GASB rule is triggering a wave of benefit cuts and premium increases by public employers. These responses support the notion that accounting rules matter and that voter information is incomplete.

Anecdotal evidence suggests that management of pensions funds is also problematic, e.g. the recurrent scandals involving "pay to play" schemes. The monitoring problem is further complicated by investments in risky assets, which makes performance evaluation

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[5] A complication for public pension accounting is that there are unresolved legal questions. Notably, it is unclear to what extent public employees have the right to accrue future pensions. If they do, as Peng (2009) argues, and are protected from dismissal, projected benefits are a legitimate measure of liabilities, though the rate at which they accrue—say, following entry age normal, unit credit or other methods—then becomes ambiguous. U.S. accounting rules allow all these methods.

difficult. Risky assets can also lead to underfunding due to unexpected losses. If losses look like a signal of mismanagement, fund investments are likely biased towards safe assets. This raises the broader question of optimal investment strategy.

2.5. Funding Levels and Investment Strategy

Investment strategy matters for funding because risk is a key source of fluctuations in funding ratios. Even with "conservative" funding, the returns on assets and liabilities invariably differ because DB plans liabilities are contingent on variables like wage growth and survival rates, which are difficult to hedge on financial markets. In many funds, the mismatch in returns is increased intentionally by investment strategies that strive for high returns, e.g. by high portfolio shares in equities and by "alternative" investments in hedge funds, private equity, real estate, and commodities.

Finance theory teaches that high returns require taking risk. This suggests the use of safe interest rates to measure risk-adjusted returns on *all* financial assts. However, explaining the equity premium is a challenge (Mehra and Prescott 1985). Managers of public pension funds must necessarily take a stand on the explanation. If the premium is due to risk aversion—say, a crash premium a la Barro and Ursua (2009)—the use of safe interest rates is correct. If the premium is instead due to liquidity effects, investor myopia, or other non-fundamental reasons, an all-equity portfolio may promise abnormal returns. Problems of performance measurement are a complicating factor. Lo (2008) notes that many hedge funds have payoff profiles similar to short positions in far-out-of-the-money put options, i.e. seemingly abnormally high returns most of the time (a high Sharpe ratio), but with a possibility of severe losses in rare states of nature.

To avoid the problems of managing risky assets, a fund might invest in Treasury securities. However, if taxpayers face credit market frictions, their cost of funds are higher than Treasury interest rates. The question of taxpayers' funding cost has not received sufficient attention in the literature and is therefore the focus of the next section.

3. A Model of Local Government

This section examines a model of local public pensions. The model has quite a number of necessary elements—a local population, a local government, a public work force, a national or world economy that determines outside options, and a national tax system that motivates tax-sheltered savings. Moreover, the setup should not inherently favor DB over DC plans (i.e., both could be optimal under some conditions), and it should be consistent with observed fiscal institutions—notably, the balanced budget rule.

To keep the model tractable, I reduce the life cycle to three periods, abstract from demographic fluctuations, and introduce key frictions such as intermediation cost in a reduced-form fashion. The term local is used broadly here to include states as well as smaller entities such as cities or counties. A key assumption is that factor prices and national taxes are exogenous. The main model considers a local government financed by property taxes. As in Epple and Schipper (1981), public debt and pensions are capitalized in home prices.

3.1. Assumptions and Benchmark Allocation

The community is populated by overlapping generations of individuals who live for three periods. The three cohorts are interpreted as young, middle, and old age. Time is indexed by t and cohorts by age i (i=1,2,3). The community has N residents in each cohort. Three periods is the minimum needed to model essential age-earnings and pension vesting issues, while keeping model analytically tractable. The young supply one unit of labor, the middle-aged supply $e > 1$ units (capturing an age-earnings link), and the old are retired. All individuals have preferences over consumption (c_t^i), local public services (g_t), and housing services.

The outside world determines wages and returns to financial investments. Let w_t be the marginal product of a labor unit. Let financial assets be valued under a pricing kernel $m_{tn} = m(s_{t+n} \mid s_t)$; that is, payoffs $a_{t+n} = a(s_{t+n})$ in state of nature s_{t+n} in period t+n are

valued in period t at conditional expectation $E_t[m_{tn}a_{t+n}]$. Let $m_t = m_{t1}$ be the one-period pricing kernel and let $1 + r_t = 1/E_{t-1}m_t$ define the safe interest rate.[6]

The community has $\overline{N}$ identical houses that can be owned by residents or by commercial owners. Owner-occupied houses provide a consumption value $v(g_t)$ per period for the young and middle-aged, which depends in part on public services; $v(g)$ is increasing and concave. Individual utility is

$$U_t = u(c_t^1 + h_t^1 v(g_t)) + \beta u(c_{t+1}^2 + h_{t+1}^2 v(g_{t+1})) + \beta^2 u(c_{t+2}^3), \qquad (1)$$

where $\beta \in (0,1)$ captures time preference and $h_t^i \in [0,1]$ indicates home ownership. Ownership greater than one is possible but would be treated as commercial. All homeowners pay property taxes T_t.

Commercial owners earn the value $v(g_t)$ as rental income but incur a management cost of $\chi_H > 0$, so net rental income is $v(g_t) - \chi_H - T_t$. Commercial owners capitalize rental income under the pricing kernel m, which yields a house value

$$H_t = \sum_{n \geq 0} E_t\big[m_{tn}\big(v(g_{t+n}) - \chi_H - T_{t+n}\big)\big] \qquad (2)$$

Assume $\overline{N} > 2N$, so not all houses can be owner-occupied. Then (2) defines the equilibrium house price. Also assume (for simplicity) χ_H is high enough that all residents prefer to own rather than rent, so $h_t^1 = h_t^2 = 1 \forall t$. That is, the young in period t buy a house at price H_t, they pay taxes T_t and (later) T_{t+1}, and they sell the house at price H_{t+2} when old.

As simple benchmark, abstract from income taxes but allow for national old age transfers TR (social security). Then individuals maximize utility subject to the budget constraint

$$w_t - c_t^1 - T_t - H_t + E_t\big[m_{t1}(ew_{t+1} - c_{t+1}^2 - T_{t+1})\big] + E_t\big[m_{t2}(TR_{t+2} + H_{t+2} - c_{t+2}^3)\big] = 0 \quad (3)$$

The optimality conditions are

$$\frac{\beta u'(\tilde{c}_{t+1}^2(s_{t+1}))}{u'(\tilde{c}_t^1(s_t))} = m(s_{t+1} \mid s_t) \forall s_{t+1} \text{ and } \frac{\beta^2 u'(c_{t+2}^3(s_{t+2}))}{u'(\tilde{c}_t^1(s_t))} = m(s_{t+2} \mid s_t) \forall s_{t+2}, \qquad (4)$$

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[6] Many of the life-cycle arguments below employ essentially deterministic reasoning. A useful heuristic for readers not used to pricing kernels is to think of m as a discount factor like $1/(1+r)$.

where $\tilde{c} = c + v(g)$. Thus marginal utilities are aligned with the pricing kernel across periods and states of nature. Because individuals can save on their own, is no need for pension funds. Under empirically plausible assumptions, net financial assets are negative from youth to middle age and positive from middle to old age;[7] this is assumed in the following. Let $\overline{U}_t$ denote the maximized utility, which is a function of public services, home prices, and property taxes.

The task of local government is to provide public services that maximize homeowner utility $\overline{U}_t$. Assume

$$g_t = G_t / \overline{N} = G(L_t^1, L_t^2) / \overline{N} \tag{5}$$

is produced by public employees, where G is increasing, concave, and linearly homogeneous, and L_t^1 and L_t^2 are the number of young and middle-age employees.[8] Assuming the government pays market wages, labor costs are $W_t = w_t L_t^1 + e w_t L_t^2$. To minimize cost, relative productivity must match relative wages, so

$$\frac{\partial G}{\partial L^2}(l, 1-l) / \frac{\partial G}{\partial L^1}(l, 1-l) = e.$$

This defines the optimal share of young workers $l^* = L^1 / (L^1 + L^2)$. Because public debt would be neutral (capitalized in house prices), setting $T_t = W_t / \overline{N}$ is without loss of generality. Optimal public employment maximizes the value of services minus their cost, so $\partial(v(g_t) - T_t) / \partial L_t^i = 0$. This implies

$$g_t = g_t^* \equiv (v')^{-1} \left(w_t \frac{l^* + e(1 - l^*)}{\overline{N}} \right) \text{ and } T_t = T_t^* \equiv w_t g_t^* \frac{(l^* + e(1 - l^*))\overline{N}}{G(l^*, 1 - l^*)}.$$

Because $(v')^{-1}$ is decreasing, optimal services are declining in the cost of labor. The implied house price is

$$H_t^* = \sum_{n \geq 0} E_t \left[m_{tn} \left(v(g_t^*) - T_t^* - \chi_H \right) \right]. \tag{6}$$

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[7] That is, houses are costly, earnings peak in middle age, and social security and home sales are not enough to finance retirement. As rough calibration, suppose w=1, e=1.5, a period of 20 years, 3% annual return on savings of 3%, house price of 5 times annual wages, 1% annual property taxes; so H=0.25 and T=0.05 per 20 year period. Assume v=0.2, slightly above the carrying cost of a house of 0.16, TR=0.2, and 3% time preference. Then $\tilde{c}^1 = \tilde{c}^2 = c^3 \approx 1.05$, savings are -0.15 in period 1, +0.60 in period 2, and -0.60 in period 3. So individuals borrow when young and save in middle age.

[8] An interesting extension would be to consider differential productivity of "experienced" employers who worked for the same government in the previous period versus new middle age hires. This would provide a naturally motivation for career employment and a way to address the human capital issues raised in Section 2.

Assume $v(g_t^*) - T_t^* > \chi_H$, so house prices are positive.

3.2. Income Taxes and Intermediation Cost

Individuals are in reality subject to national income taxes. As simple proxy for progressive taxation, assume marginal tax rates τ^i are constant but age-specific. Because the age-earnings profile peaks in middle age, assume $\tau^2 > \tau^1$ and $\tau^2 > \tau^3$. Taxes apply to wage income minus pension contributions, to returns on regular (taxable) investments, and to pension payouts.

Income taxes provide a strong motivation for pension plans.[9] Because the setup cost of a pension plan is negligible compared to the tax benefits, all employers will offer at least a DC pension plan. With competition between employers, all tax savings accrue to employees, so total compensation w_t remains unchanged. Because employer contributions would reduce cash wages one for one, assume (w.l.o.g.) that DC plans have zero employer contributions.

Regular investments, debt, and pensions now require separate accounting. Let $x_t^i \geq 0$ denote retirement contributions, let r_{xt+1}^i be the returns, and let

$$X_{t+2} = [x_t^1(1 + r_{xt+1}^1) + x_{t+1}^2](1 + r_{xt+2}^2). \tag{7}$$

be pension payouts. Payouts in middle age are prohibited. Individuals may also hold regular taxable assets $a_t^i \geq 0$ with returns r_{at+1}^i and borrow amounts $d_t^i \geq 0$.

For investments, assume the distribution of returns is chosen by investors subject to the no-arbitrage conditions $E_t[m_{t+1}(1 + r_{xt+1}^i)] = 1 - \chi_{DC}$ and $E_t[m_{t+1}(1 + r_{at+1}^i)] = 1$, i=1,2, where $\chi_{DC} \geq 0$ allows for a (small) cost of managing assets. That is, investments can be interpreted as portfolios of Arrow securities that provide payoffs in specific states of nature. This complete markets setting—apart from cost—provides a straightforward accounting for uncertainty.[10]

For individual debt, assume lenders incur intermediation costs that increase with the level of debt (say, for screening or monitoring). Borrowers rank sources of debt by borrowing

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[9] Individual IRA accounts or large tax exemptions for capital income would deliver similar tax benefits, but U.S. tax laws allow greater contributions in employer plans. IRAs can be subsumed under DC plans.

[10] Missing markets are discussed below. Management cost on regular savings are moot because they will be return-dominated even without cost.

cost minus tax benefits. To express this parsimoniously, let $\chi_d(\hat{d})$ denote the net cost at debt level $\hat{d}$ and $\bar{\chi}_d(d_t^i) = \int_0^{d_t^i} \chi_d(\hat{d}) d\hat{d}$ the total cost of debt. For symmetry with assets, assume borrowers choose the return distribution, which can be written as $E_t[m_{t+1}(1 + r_{dt+1}(\hat{d}))] = 1 + \chi_d(\hat{d})$ on the margin and $E_t[m_{t+1}(1 + \bar{r}_{dt+1}(d_t^i))] = 1 + \bar{\chi}_d(d_t^i)$ for total debt. Net cost may be negative for tax-deductible debt (e.g. mortgages). To limit arbitrage, assume marginal costs approach infinity at some (unspecified) maximum debt. To avoid corner solutions, assume $\chi_d(0) \le 0$.

With these financing choices, individuals maximize utility subject to the budget equations

$$c_t^1 + h_t^1 T_t = (w_t - x_t^1)(1 - \tau^1) + d_t^1 - a_t^1 - h_t^1 H_t \tag{8}$$

$$\begin{aligned} c_{t+1}^2 + h_{t+1}^2 T_{t+1} = {}& (ew_{t+1} - x_{t+1}^2)(1 - \tau^2) + d_{t+1}^2 - [1 + \bar{r}_{dt+1}(d_t^1)]d_t^1 \\ & + (h_t^1 - h_{t+1}^2)H_{t+1} - a_{t+1}^2 + [1 + (1 - \tau^2)r_{at+1}^1]a_t^1 \end{aligned} \tag{9}$$

$$c_{t+2}^3 = TR_{t+2} + X_{t+2}(1 - \tau^3) + h_{t+1}^2 H_{t+2} - [1 + \bar{r}_{dt+1}(d_{t+1}^2)]d_{t+1}^2 + [1 + (1 - \tau^3)r_{at+2}^2]a_{t+1}^2. \tag{10}$$

Proceeding recursively, the first order conditions in middle age for pensions, taxable assets, and debt are

$$\frac{\partial U}{\partial x_{t+1}^2} = -\beta u'(\tilde{c}_{t+1}^2)(1 - \tau^2) + E_{t+1}\left[\beta^2 u'(c_{t+2}^3)(1 + r_{xt+2}^2)\right](1 - \tau^3) + \mu(x_{t+1}^2) = 0, \tag{11}$$

$$\frac{\partial U}{\partial a_{t+1}^2} = -\beta u'(\tilde{c}_{t+1}^2) + E_{t+1}\left[\beta^2 u'(c_{t+2}^3)(1 + (1 - \tau^3)r_{at+2}^2)\right] + \mu(a_{t+1}^2) = 0 \tag{12}$$

$$\frac{\partial U}{\partial d_{t+1}^2} = \beta u'(\tilde{c}_{t+1}^2) - E_{t+1}\left[\beta^2 u'(c_{t+2}^3)(1 + r_{dt+2}^2)\right] + \mu(d_{t+1}^2) = 0 \tag{13}$$

where $\mu(z) \ge 0$ denotes the Kuhn-Tucker multiplier for any constraint $z \ge 0$.

Consider first the case $x_{t+1}^2 > 0$. From (11) and the arbitrage condition for returns, one obtains

$$E_{t+1}\left[\left(\frac{\beta u'(c_{t+2}^3)}{u'(\tilde{c}_{t+1}^2)}\Theta_{x2} - m_{t+2}\right)(1 + r_{xt+2}^2)\right] = 0 \text{ with } \Theta_{x2} \equiv \frac{1 - \tau^3}{1 - \tau^2}(1 - \chi_{DC}). \tag{14}$$

Because this condition must hold for all investments, including Arrow securities, the marginal rate of substitution in all states of nature must satisfy

$$\frac{\beta u'(c_{t+2}^3(s_{t+2}))}{u'(\tilde{c}_{t+1}^2(s_{t+1}))} = \frac{1}{\Theta_{x2}} m(s_{t+2} \mid s_{t+1}) \quad \forall s_{t+2} \tag{15}$$

This is similar to (4), but distorted by the tax factor Θ_{x2}. Recall that $(1-\tau^3)/(1-\tau^2) > 1$ and assume χ_{DC} is small. Then $\Theta_{x2} > 1$. This means federal taxes provide a "boost" to pension returns and a strong incentive to contribution in middle age.

If (15) holds, (12) implies $\mu(a_{t+1}^2) > 0$, so $a_{t+1}^2 = 0$. Regular savings are inferior to pensions. In addition, (15) combined with (13) imply

$$\chi_d(d_{t+1}^2) = \Theta_{x2} - 1 > 0. \tag{16}$$

This defines an optimal debt level $d_{t+1}^{2*} > 0$. Intuitively, individuals borrow to fund pensions up to the point where the pension tax incentives in Θ_{x2} are offset by intermediation cost. Hence Θ_{x2} can also be interpreted as borrowing cost: $\Theta_{x2} = 1 + \chi_d(d_{t+1}^2)$.

For completeness, consider the possibility that $x_{t+1}^2 = 0$. Then analogous arguments imply $\dfrac{\beta u'(c_{t+2}^3(s_{t+2}))}{u'(\tilde{c}_{t+1}^2(s_{t+1}))} \le m(s_{t+2}\mid s_{t+1})/\Theta_{x2}$, which in turn implies $a_{t+1}^2 = 0$ and $d_{t+1}^2 \ge d_{t+1}^{2*} > 0$. Given $d_{t+1}^2 > 0$, (13) implies $\dfrac{\beta u'(c_{t+2}^3(s_{t+2}))}{u'(\tilde{c}_{t+1}^2(s_{t+1}))} = m(s_{t+2}\mid s_{t+1})/\Theta(d_{t+1}^2)$, where $\Theta(d_{t+1}^2) = 1/(1 + \chi(d_{t+1}^2) \le \Theta_{x2}$. Thus the marginal rate of substitution would be pushed down even more than in the $x_{t+1}^2 > 0$ case.

To summarize: in all cases, *income taxes imply a downward distortion in the marginal rate of substitution between peak earnings years and retirement*. In the following, to avoid distracting case distinctions, assume the case $x_{t+1}^2 \equiv x_{DC}^2 > 0$ applies.

For the young, similar first order conditions as in middle age apply for regular assets and debt (not shown). The condition for optimal pension contributions is different because pensions must be held to retirement:

$$\frac{\partial U}{\partial x_t^1} = -u'(\tilde{c}_t^1)(1-\tau^1) + E_t\left[\beta^2 u'(\tilde{c}_{t+2}^3)(1+r_{xt+1}^1)(1+r_{xt+2}^2)\right](1-\tau^3) + \mu(x_t^1) = 0. \tag{17}$$

Using (15) to replace $u'(\tilde{c}_{t+2}^3)$, this implies

$$E_t\left[\left(\frac{\beta u'(\tilde{c}_{t+1}^2)}{u'(\tilde{c}_t^1)}\Theta_{x1} - m_{t+1}\right)(1+r_{xt+1}^1)\right] + \frac{\mu(x_t^1)}{u'(\tilde{c}_t^1)(1-\tau^1)} = 0$$

$$\text{where } \Theta_{x1} = \frac{1-\tau^2}{1-\tau^1}(1-\chi_{DC})^2 \le \frac{1-\tau^2}{1-\tau^1} < 1. \tag{18}$$

In young age, the tax factor $\Theta_{x1} < 1$ discourages retirement contributions.

Life-cycle arguments suggest that the young have strong incentives to borrow. To avoid distracting case distinctions, assume $d_t^1 > 0$ and $\chi_d(d_t^1) > 0$. Then the first-order condition for debt has a zero Kuhn-Tucker multiplier and can be written as

$$E_t\left[\left(\frac{\beta u'(\tilde{c}_{t+1}^2)}{u'(\tilde{c}_t^1)}(1 + \chi_d(d_t^1)) - m_{t+1}\right)(1 + r_{dt+1})\right] = 0, \tag{19}$$

with implies

$$\frac{\beta u'(\tilde{c}_{t+1}^2(s_{t+1}))}{u'(\tilde{c}_t^1(s_t))} = \frac{1}{1 + \chi_d(d_t^1)} m(s_{t+1} \mid s_t) \quad \forall s_{t+1}. \tag{20}$$

Optimal debt aligns the marginal rate of substitution with the pricing kernel, now distorted downwards by intermediation cost. Combining (20) and (18), $\mu(x_t^1) > 0$ follows from $\Theta_{x1} < 1 \leq 1 + \chi_d(d_t^1)$, so $x_t^1 = x_{DC}^1 = 0$. Similar reasoning implies $a_t^1 = 0$.

The model suggests that the young and middle-aged hold debt. For empirical support, Table 1 shows data on the prevalence of borrowing and on the cost of borrowing. According to the Survey of Consumer Finances 2007 (SCF), more than 80% of families with head of household under 65 hold debt. Mortgages are held by majorities in the 35-64 age brackets, installment debt by majorities in the under-54 age brackets, and credit card debt by majorities in the 35-54 age brackets. Thus the vast majority of U.S. families are debtors, and for most of them (all but prime mortgage debtors who itemize), debt is costly.

The model also implies that young borrowers should not contribute to pension plans. Empirically, many young workers indeed contribute nothing or very little, which is sometimes viewed as puzzling. According to SCF, 58% of families under 35 have no retirement accounts at all (see Table 1). Retirement contributions by the young could be rationalized in model extensions (see below), but given the data, zero contributions are a useful benchmark.

In summary, the model yields a simply and empirically plausible life cycle. The young work, buy a house, and borrow. The middle-aged save for retirement and use debt only for tax reasons. The old consume and liquidate their assets.[11]

Marginal rates of substitution are given by (15) between middle and old age and by (20) between young and middle age. Importantly, both marginal rates of substitution are less

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[11] One could easily add a joy-of-giving bequest motive to the model (e.g. for giving the house to the kids), so liquidation should not be taken literally.

than the pricing kernel. The relationship between Θ_{x2} in (15) and $1 + \chi_d(d_t^1)$ in (20) is an empirical issue. As a rough calibration, a tax wedge between $\tau^2 = 30\%$ and $\tau^3 = 15\%$ over 20 years would imply an annual tax advantage of about 1%; $(1 - \tau^2)/(1 - \tau^3) = 0.85/.70 = (1 + 0.98\%)^{20}$. A 25bp management cost (using index funds) would implied a 75bp annualized gain in Θ_{x2}. Spreads between T-bills and credit card rates are about 14% and spreads on car loans (the most common installment loan) about 6%; see Table 1. To calibrate $\chi_d(d_t^1)$, one would have to adjust for risk, which is difficult. An alternative proxy is the spread between the Prime Rate and T-bills, which is about 3%. Overall, the data indicate that most young families face borrowing cost of several percentage points; this suggest $1 + \chi_d(d_t^1) \geq \Theta_{x2} > 1$.

Turning to local public finance, pension incentives and costly debt have a key implication: *Most taxpayers face a marginal cost of funds greater than the rate of return on financial assets*.

3.3. Public Pensions and Public Debt

Consider the local government's financing choices with public debt and DB pensions. Because DB plans can be underfunded, they allow a form of debt.

Let D_t denote end-of-period debt. With DB pensions, let w_t^1 and w_t^2 denote period-t "cash" wages paid to young and middle-aged public employees, and let P_t denote employer contributions to the DB plan. Then $\tilde{W}_t = w_t^1 L_t^1 + w_t^2 L_t^2$ is the current wage cost. The period-t budget identity is

$$D_t = (1 + r_{Dt})D_{t-1} + \tilde{W}_t + P_t - \bar{N} \cdot T_t \tag{21}$$

The interest r_{Dt} may include an intermediation cost χ_D (constant for simplicity), so $E_{t-1}[m_t(1 + r_{Dt})] = 1 + \chi_D \geq 1$. Assuming investors impose the transversality/No Ponzi condition $E_t[m_{tn}D_{t+n}] \to 0$ as $n \to \infty$, one obtains the intertemporal budget constraint (IBC)

$$D_t^* = \sum_{n \geq 0} E_t\left[m_{tn}\left(\bar{N} \cdot T_{t+n} - \tilde{W}_{t+n} - P_{t+n} - \chi_D D_{t+n}\right)\right] \tag{22}$$

where $D_t^* = (1 + r_{Dt})D_{t-1}$ is the start-of-period debt.

Let pension fund assets F_t earns returns r_{Ft}, where $E_{t-1}[m_t(1+r_{Ft})]=1-\chi_F$ may be subject to a management cost $\chi_F \geq 0$. All pension benefits B_t are paid from the fund, either from fund assets or pay-as-you go employer contributions. The budget identity is

$$F_t = (1+r_{Ft})F_{t-1} + P_t - B_t. \tag{23}$$

The limit condition $E_t[m_{tn}F_{t+n}] \to 0$ as $n \to \infty$, implies the IBC

$$F_t^* = \sum_{n \geq 0} E_t[m_{tn}(B_{t+n} - P_{t+n} + \chi_F F_{t+n})] \tag{24}$$

where $F_t^* = (1+r_{Ft})F_{t-1}$ is start-of-period funding position. Combining (22) and (24), the present value of taxes is

$$\bar{N} \cdot \sum_{n \geq 0} E_t[m_{tn}T_{t+n}] = D_t^* + B_t - F_t^* + \sum_{n \geq 0} E_t[m_{tn}(W_{t+n} + \chi_F F_{t+n} + \chi_D D_{t+n})] \tag{25}$$

where $W_t = \tilde{W}_t + E_t[m_{t+1}B_{t+1}]$ measures total compensation (current wages and present value of benefits). Thus taxes depend on debt, promised benefits minus funding, plus future compensation and intermediation cost.

Voters care about taxes because they affect house prices. Combining (2) and (25),

$$\begin{aligned} H_t = &\sum_{n \geq 0} E_t\left[m_{tn}\left(v(g_{t+n}) - \chi_H - \tfrac{1}{N}W_{t+n}\right)\right] \\ &- \tfrac{1}{N}\left(D_t^* + B_t - F_t^*\right) - \tfrac{1}{N}\sum_{n \geq 0} E_t[m_{tn}(\chi_F F_{t+n} + \chi_D D_{t+n})] \end{aligned} \tag{26}$$

The first line in (26) reflects the value of public services and their real cost; the second line reflects financing choices.

To attract employees, an employment package with DB plan must be competitive with DC plans; and with complete markets, the optimal DB plan must replicate DC contributions and benefits. Instead of paying workers w_t and ew_{t+1} in young and middle age, the government pays (in general) $w_t^1 = w_t - x_t^1$ and $w_{t+1}^2 = ew_{t+1} - x_{t+1}^2$ and it promises retired workers a pension X_{t+2}, so $B_{t+2} = X_{t+2}L_{t+1}^2$. If $x_{DC}^1 = 0$, as suggested in the previous section, the DB plan simplifies: young employees receive an unreduced salary $w_t^1 = w_t$ and no promise of benefits.[12] Then from (7),

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[12] In practice, legal restrictions may prevent DB plan sponsors from excluding the young. However, vesting requirements could have a similar effect, suggesting an efficiency argument for long vesting periods. The optimality of zero contributions in young age suggests caution in using accounting methods such as entry-age-normal that mechanically allocate projected

$$W_t = \tilde{W}_t + E_t[m_{t+1}B_{t+1}] = w_t L_t^1 + \left(ew_t - \chi_{DC} x_{DC}^2\right)L_t^2. \tag{27}$$

This means that to the extent that managing DC investment is costly, a DB plan reduces employment cost. Compared to Section 3.1, the reduced cost $ew_t - \chi_{DC} x_t^2$ of middle-age employees shifts the optimal employment mix ($L_t^1/(L_t^1 + L_t^2) < l^*$) and increases the optimal scale of public services ($g_t > g_t^*$). For low DC management cost ($\chi_{DC} \approx 0$), $g_t \approx g_t^*$ and line 1 of (26) is approximately equal H_t^*, so the "real" allocation is essentially unchanged.

Line 2 of (26) shows that initial debt, pension promises, and pension fund assets are fully capitalized. Because of intermediation cost, expected future debt reduces house values. If DB fund management is costly ($\chi_F > 0$), future fund balances also reduce house values. Intermediation costs discourage both debt and assets.

Recall that most voters—resident homeowners—are debtors. While real estate investors capitalize houses at the pricing kernel m, voters discount the future at the marginal rates of substitution (15) and (20), which are strictly greater. Because $\Theta_{x2} = 1 + \chi_d(d)$, both marginal rate of substitution are based on borrowing cost and can be written generically as $m_{t+1}/[1 + \chi_d(d)]$. Assuming votes takes place every period after homes are traded, middle-aged individuals vote to maximize

$$\eta_t^2 \equiv \left(v(g_t) - T_t\right) + E_t[\frac{m_{t+1}}{1 + \chi_d(d_t^2)} H_{t+1}]. \tag{28}$$

The young vote to maximize

$$\eta_t^1 \equiv v(g_t) - T_t + E_t\left[\frac{m_{t+1}}{1 + \chi_d(d_t^1)}\left(v(g_{t+1}) - T_{t+1} + \frac{m_{t+2}}{1 + \chi_d(d_{t+1}^2)} H_{t+2}\right)\right]. \tag{29}$$

If $\chi_D < \chi_d(d_t^1)$ and $\chi_D < \chi_d(d_t^2)$, both generations benefit from a debt-financed tax cut in period t that is reversed in period t+1.[13] The old are indifferent because they have sold their houses.

Because the χ_D-term in (26) is negative, debt in future periods reduces house values. Hence all voters would favor a rule that outlaws debt as soon as they reach old age. If voting

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benefits over an entire career. The interpretation of W_t as total compensation implicitly exploits a single period of contributions; with multi-period accrual, more elaborate notation would be needed.

[13] The literature has focused on tax distortions as reasons why tax timing matters. Tax distortions could be added (say, a collection cost) but they would not add much insight because tax smoothing would not be optimal.

is sequential, rational investors should expect persistent debt in the future and value houses accordingly. But voters will support a balanced budget rule—a once and for all decision to outlaw debt—provided the impact of persistent debt on home prices (the χ_D-term) outweighs the current gains from $\chi_d(d) - \chi_D$. Hence the model is consistent with popular support for balanced budget rules.[14] In the following, assume a balanced budget rule applies so $D_t \equiv 0$.

Importantly, support for balanced budgets does *not* imply support for pension funding. Even with zero funding cost ($\chi_F = 0$), strictly prefer minimal funding, which means $F_t = 0$. (As proof, suppose funding was positive. Then the young and middle-aged would benefit from tax cut in period t that is reversed in period t+1 and financed by a delay in employer pension contributions.) Note that if full funding were mandated by an outside regulator (e.g. GASB), it would cancel out essentially all welfare gains from offering a DB instead of a DC public pension plan (all but the minor gains from $\chi_F < \chi_{DC}$).

Young voters may even favor a more extreme policy. If $\chi_d(d_t^1) > \chi_d(d_t^2)$, they may vote to give middle-aged public employees a benefit package with reduced wages and higher benefits, $w_t^2 < ew_t - x_{DC}^2$ and $x_t^2 > x_{DC}^2$. Compared to the private sector, such benefits would appear "bloated." In effect, young voters would borrow from middle-aged public employees at cost $\chi_d(d_t^2)$, which is less than their own cost of borrowing. Bloated benefits would reduce house prices, however. Hence the middle-aged would be strictly opposed, and the young approve only if the gains from $\chi_d(d_t^1) - \chi_d(d_t^2)$ outweigh the negative price effect. Thus bloated benefits are possible but not a prediction of the model.

The robust conclusion is a strong case against pension funding, and this in a model that supports the balanced budget rule.

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[14] The exact conditions are complicated but seem mild. For example, the middle aged to support a permanent debt reduction provided $\chi_d(d_t^2) - \chi_D \leq \chi_D / r_\infty$ where r_∞ is the interest rate on a consol. Say, if r_∞=5%, the condition holds unless $\chi_d(d_t^2)$ is 21-times greater than χ_D. Also, one could show that the model is consistent with tax-exempt borrowing for capital projects, provided the tax subsidy covers the intermediation cost. The tax-exempt debt would be capitalized as if $\chi_D < 0$, which means property values would be increased. Note that because $\eta_t^1 = (v(g_t) - T_t) + E_t[\eta_{t+1}^2 \cdot m_{t+1} / (1 + \chi_d(d_t^1))]$, any proposal to defer taxes that is favored by the middle-aged (so η_{t+1}^2>0) is also supported by the young. Thus the median voter in this model is best interpreted as a median-age homeowner.

3.4. Accounting Implications

Suppose the optimal public employment contract includes a DB plan that replicates DC savings. How should government account for the plan obligations?

Specifically, suppose middle-age government workers receive a wage $w_t^2 = ew_t - x_t^2$ in the current period and are promised $X_{t+1} = x_t^2(1 + r_{xt+1}^2)$ in retirement. These benefits may be unfunded or (partially) backed by a fund F_t earning returns r_{Ft+1}. Young employees receive a wage $w_t^1 = w_t$ and no immediate entitlement. Disregard management cost for clarity.

Taxpayers and investors are not unanimous how to value such pension obligations. Outside investors apply the pricing kernel m and compute

$$AAL_t^m = E_t[m_{t+1}B_{t+1}] = E_t[m_{t+1}(1 + r_{xt+1})]x_t^2 L_t^2 = x_t^2 L_t^2.$$

Taxpayers use a marginal rate of substitution based on (15) or (20), $m_{t+1}/[1 + \chi_d(d)]$. This implies

$$AAL_t = E_t\left[\frac{m_{t+1}}{1+\chi_d(d)}X_{t+1}L_t^2\right] = \frac{1}{1+\chi_d(d)}x_t^2 L_t^2 < AAL_t^m.$$

For all borrowers, discounting by the pricing kernel would overstate the opportunity cost of pension promises. Proper discounting reflects the risk-characteristics of *liabilities*, as in standard finance theory. However, discount rates should include an allowance for intermediation cost, i.e., it should be derived from *borrowing cost*, not from investment returns. As result, accountants may have a point when they use discount rates greater than Treasury interest rates—though not for the right reasons.

A complication is that borrowing rates generally differ across individuals, as exemplified by age differences in the model. With heterogeneous borrowing cost, a single discount rate is a mere convention. As simple adjustment for intermediation cost, one might add a fixed spread to investment-based discount rates. For example, "safe" pension benefits are commonly discounted at duration-matched Treasury rates (Novy-Marx and Rauh 2009). Over the last 20 years, 10-year Treasuries yields have averaged 2.8% in real terms (5.6% nominal, 2.8% inflation). If the median voter faces an intermediation cost of 1.5-3%, one

obtains real discount rates of 4%-6% and nominal discount rates around 7-9%. This is close to values commonly used by actuaries. (Currently Treasury yields are lower.)

A further complication is that because funding is suboptimal, measures of "underfunding" are difficult to interpret. Funding equal to AAL, $F_t / AAL_t = 100\%$, could be labeled "full" funding. However, because fund returns are less than taxpayers' borrowing cost, "full" funding in this sense would leave an expected underfunding in the next period:

$$UAAL_t = E_t\left[\frac{m_{t+1}}{1+\chi_d(d)}\left(B_{t+1} - F_{t+1}^*\right)\right] = \frac{1}{1+\chi_d(d)}(x_t^2 L_t^2 - F_t) = AAL_t - \frac{F_t}{1+\chi_d(d)}, \qquad (30)$$

which is positive for all $F_t < x_t^2 L_t^2$. A possible way to account for these return differences would be to discount funded obligations by risk-adjusted asset returns and unfunded obligations by (higher) risk-adjusted debt returns.[15]

4. Legal Ambiguity and Default Risk

Legal ambiguity and default risks deserve attention because they have first order effects on optimal funding.

By legal ambiguity I mean the risk that the plan sponsor can successfully dispute the scope or existence of pension obligations. By default risk I mean the risk that the plan sponsor will default outright on its liabilities—presumably on debt as well as pensions. A key distinction is that default is usually observable and unambiguous. Hence default should be insurable, whereas legal ambiguities are not. (Hence I prefer the term legal *ambiguity* instead of legal *risk*.) Almost by construction, if a plan sponsor can find a legal flaw or otherwise challenges a pension payment, an insurer could raise the same objections. Employees must always worry how the courts might rule, and this risk is uninsurable.

For example, consider a plan that promises a fixed percentage of final salary, which is then indexed to inflation, as is common in DB plans. Final salary is an ambiguous term when the structure of compensation is evolving. A generous definition would invite attempts to manipulate the timing of payments to show an abnormally high final salary (a.k.a. "pension

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[15] Note that GASB rules for retiree health care prescribe exactly the opposite, higher discount rates on funded liabilities than on unfunded ones; this seems unjustified.

spiking"). A stingy definition—excluding all bonus-type payments—would allow the employer to cut pensions by reducing base salaries and instead paying repeated bonuses. (With some inflation, freezing nominal salaries would work similarly.) Indexing may cause trouble if simple formulas are used that, over time, become so "unreasonable" that courts may not enforce them.

Funding may then serve as collateral and provides a floor on benefits. A key question is to what extent pension assets are reserved for pension benefits, or if plan sponsor can reclaim assets under some conditions. Funding is only effective as collateral if it cannot be diverted. Strong regal restrictions against "raids" by plan sponsors are indeed widespread, even in cases when a plan seems vastly overfunded. Such restrictions are difficult to rationalize without legal ambiguity.[16] Moreover, the governance structure of many funds is designed to protect beneficiaries.

To model legal ambiguity, assume each "macro" state of nature (s_{t+1}) has two possible realizations of legal risk (s^-_{t+1}, s^+_{t+1}). With probability $\bar{\pi}$, the promised benefits $X(s_{t+1})$ are disputed and the plan sponsor successfully refuses additional contributions. Then actual payments are $X(s^-_{t+1}) = \min\{X(s_{t+1}), F^*(s_{t+1})/L^2_t\}$. With probability $1 - \bar{\pi}$, promised benefits are honored, so $X(s^+_{t+1}) = X(s_{t+1})$. Note that the same payoffs would occur in case of default; hence the model applies analogously to default risk.[17]

As a behavioral extension, denote employees' perception of legal ambiguity by $\bar{\pi}^e$ and let it be distinct from the employer's view. For simplicity, assume that $\bar{\pi}$ and $\bar{\pi}^e$ are exogenous and constant. Also assume the median voter is middle aged, so the public employer has a well-defined objective, to maximize η^2_t.

The economic implication of legal ambiguity is an uninsurable risk. Retirement consumption is $c^3_{t+1}(s^+) = TR_{t+1} + H_{t+1} + X(s_{t+1})$ if the employer pays as promised.

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[16] Otherwise it would be economically efficient to give the plan sponsor, acting as residual claimant, a right to recover overfunding. But with legal uncertainty, the obligations are part of the dispute and hence "overfunding" is not well defined. A right to recover funds would undermine the collateral function.

[17] I focus on legal ambiguity in part for brevity (as the application to default risk is technically analogous) and in part because defaults raise the question why fund managers or beneficiaries are not taking out insurance policies against a default of the plan sponsor.

Retirement consumption is $c_{t+1}^3(s^-) = TR_{t+1} + H_{t+1} + F^*(s_{t+1})/L_t^2$ if employees share the fund assets. Whenever the funding ratio $f \equiv F^*(s_{t+1})/(X(s_{t+1})L_t^2)$ is less than 100%, default implies a discrete downward jump in consumption and an upward jump in marginal utility. The jump is increasing in the ratio of promised pension to retirement income, in the degree of underfunding, and relative risk aversion (denoted γ):

$$\Delta u'(f) \equiv \frac{u'(c_{t+2}^3(s^-)) - u'(c_{t+2}^3(s^+))}{u'(c_{t+2}^3(s^+))} = \left(1 - (1-f)\frac{X(s_{t+1})}{c_{t+2}^3(s^+)}\right)^{-\gamma} - 1 \geq 0.$$

The ratio $X(s_{t+1})/c_{t+1}^3(s^+)$ can be called *pension dependence*. Note that the relevant funding ratio is the ratio of assets to obligations in the retirement period.[18]

Optimal pension design then a choice of funded benefits, unfunded benefits, and a wage w_t^2 that maximizes η_t^2 subject to the constraint that the employment package gives workers at least the same utility as a private job with DC pension. It is straightforward to show that an interior solution ($0 < f < 1$) requires that

$$1 + \frac{\chi_F + \chi_d(d_t^2)}{1 - \chi_F} = \frac{1 - \bar{\pi}}{1 - \bar{\pi}^e}\left[1 + \bar{\pi}^e \cdot \Delta u'(f)\right]. \tag{31}$$

Whenever the l.h.s. is greater, zero funding is optimal. If the r.h.s. were greater, full funding would be optimal. The l.h.s. is greater than one and interpretable as the cost to taxpayers of a funded as compared to an unfunded pension. The r.h.s. has two parts. The ratio $(1-\bar{\pi})/(1-\bar{\pi}^e)$ is a misperceptions term that exceeds one if employees' confidence in the plan is less than the employer's. The bracketed term $1 + \bar{\pi}^e \Delta u'(f)$ exceeds one if employees perceive a probability of default ($\bar{\pi}^e > 0$) and if this risk is uninsured ($\Delta u'(f) > 0$).

Specific numbers may help build an intuition. As rough calibration, assume a 50bp annual funding cost and a 1.5% borrowing cost. Then $\chi_F \approx 10\%$ over 20 years, $\chi_d(d_t^2) \approx 35\%$, and $\frac{\chi_F + \chi_d(d_t^2)}{1 - \chi_F} \approx 0.5$. Assume symmetric expectations ($\bar{\pi} = \bar{\pi}^e$) and $\gamma = 4$. Suppose the employer pension is half of planned retirement income. Then an unfunded pension would imply $u'(c_{t+2}^3(s^-))/u'(c_{t+2}^3(s^+)) \approx 0.5^{-\gamma} = 16$, so $\Delta u'(0) \approx 15$. Equality in (31)

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[18] The definition of *f* here is consistent with the definition of underfunding in eq. (30): *f* also equal UAAL divided by the present value of benefits discounted at the taxpayers' cost of funds. Note that the analysis also applies without borrowing cost. For $\chi_d = 0$, *f* is also the ratio of current assets to promised benefits discounted at the regular pricing kernel *m*.

would require a risk of $\bar{\pi} = 0.5/15 \approx 3.4\%$. If $\bar{\pi} \leq 3.4\%$, zero funding is optimal. If $\bar{\pi} > 3.4\%$ partial funding is optimal. For example, if $\bar{\pi} = 25\%$, (31) prescribes $\Delta u'(f) = \frac{\chi_F + \chi_d(d_t^2)}{1-\chi_F}/\bar{\pi} = 0.5/0.25 = 2$, which implies $\frac{c_{t+2}^3(s^-)}{c_{t+2}^3(s^+)} = (1 + \bar{\pi}\Delta u'(f))^{-1/\gamma} \approx 0.76$.

With 50% pension dependence, this requires $f = 52\%$.

Table 2 shows results of similar calculations for a range of parameters. Generally, high pension dependence, high risk aversion, and a high degree of legal ambiguity (or default risk) favor funding, whereas intermediation costs discourage funding. In panel (a) shows cutoff probabilities for zero funding. Value are over 10% for all scenarios with 25% pensions dependence and for many of the 50%-dependence scenarios with low risk aversion and/or high cost. Thus zero funding remains optimal unless legal risk is substantial. Panel (b) shows optimal funding ratios. To avoid too many zeros, most scenarios assume relatively low cost and high risk-aversion. Many funding ratios are nonetheless under 50%. Funding ratios greater than 80% are optimal only with low cost (col.1-3) *and* high risk aversion (γ=10) *and* relatively high pension dependence ($\geq$50%).

Three general results should be noted: First, *full funding is never optimal under symmetric beliefs*. This is because $\Delta u'(f) \to 0$ as the funding ratio approaches 100%. The optimal funding ratio is always bounded away from one, but not from zero. With symmetric beliefs, zero funding is optimal whenever

$$\bar{\pi}\Delta u' \leq \frac{\chi_F + \chi_d(d_t^2)}{1-\chi_F}. \tag{32}$$

Second, even with asymmetric beliefs, *full funding is suboptimal unless employees are much more pessimistic than the employer*. Specifically, full funding is optimal if and only if

$$\frac{\bar{\pi}^e - \bar{\pi}}{1-\bar{\pi}^e} \geq \frac{\chi_F + \chi_d(d_t^2)}{1-\chi_F} \tag{33}$$

For example, if $\frac{\chi_F + \chi_d(d_t^2)}{1-\chi_F} \approx 0.5$ and the true default probability is zero, the perceived risk must be at least 33%. Otherwise optimal funding is at most partial.

Third, *legal ambiguity is always costly for the plan sponsor*. Even if the actual or perceived risks are low enough that zero funding remains optimal, doubtful promises are

discounted. An employer offering a DB plan must give workers the same utility as employers offering DC plans. Because ambiguity about defined benefits imposes idiosyncratic risk on employees, the employer must compensate with a higher wage. Funding can reduce these concerns by serving as collateral, but also at a cost.

Finally, if $\bar{\pi}$ is near one, one might consider a more radical reinterpretation. A DB plan would then provide an investment-backed retirement income just like a DC plan, except with centralized asset management and with a slight upside chance that the employer might be nice enough to pay extra benefits. However, this case would raise many new questions (beyond the scope of this paper), e.g., about the meaning of "defined" plans when legal ambiguity is so pervasive that legal definitions are almost meaningless.

5. A Model with General Taxes

The previous analysis relied on the capitalization of property taxes. For this section, assume the government imposes general taxes, as lump sum but subject to mobility restrictions.

For most local governments, labor and financial assets are highly mobile, so property is the natural base of local taxation. Nonetheless, many states (and some cities) impose taxes on income, sales, and various other activities. The model abstracts from standard excess burden issues and focuses on the exit option as the most critical constraint on taxation. Because exit is more difficult from large jurisdictions, the model of this section is best applicable to a state or large city.

To model mobility parsimoniously, assume individuals leave the state if taxes are greater than the value of public services plus an age-specific mobility cost M^i. Assume public services enter directly into utility in all periods, so

$$U_t = \sum_{i=1}^{3} \beta^{i-1} u\left(c_{t+i}^i + v(g_{t+i})\right),$$

where $g_t = G_t/(3N)$ is the public service per resident. Taxes per resident are $T_t^i \geq 0$, possibly differentiated by age. Average revenues are $T_t = (T_t^1 + T_t^2 + T_t^3)/3$. Re-define $\bar{N} = 3N$. Then the fiscal calculations in Section 3 apply analogously, with *per house* value replaced by *per*

resident. Notably, g_t^* is the optimal spending under benchmark assumptions, and the intertemporal budget constraints (22) and (24) apply.

In this model, there is no capitalization argument to discourage debt. For example, consider a debt-financed reduction in T_t^1 followed by an increase in T_{t+1}^2 by $(1+r_{Dt+1})$ times the tax cut. The impact on generation-t utility is

$$\frac{\partial U_t}{\partial(-T_t^1)} = u'(\tilde{c}_t^1) - E_t[\beta u'(\tilde{c}_{t+1}^2)(1+r_{Dt+1})] = \frac{\chi_d(d_t^1)-\chi_D}{1+\chi_d(d_t^1)} u'(\tilde{c}_t^1) > 0,$$

provided $\chi_d(d_t^1) > \chi_D$. Similarly, a debt-financed reduction in T_t^2 followed by an increase in T_{t+1}^3 by $(1+r_{Dt+1})$ raises the utility of middle-aged individuals provided $\chi_d(d_t^2) > \chi_D$. All generations' utility would be increased by tax cuts financed by taxes on future generations. Thus there is no support for a balanced budget rule.

By the same reasoning, all residents favor unfunded public retirement plans. Unfunded pension obligations are valued at residents' marginal rates of substitution, which implies higher discount rates than under the pricing kernel.

Mobility is only constraint on debt and on pension underfunding. The old leave unless $T_t^3 \leq v(g_t^*) + M^3$; the middle aged leave unless

$$T_t^2 + E_t[\frac{\beta u'(\tilde{c}_{t+1}^3)}{u'(\tilde{c}_t^2)} T_{t+1}^3] \leq v(g_t^*) + E_t[\frac{\beta u'(c_{t+1}^3)}{u'(\tilde{c}_t^2)} v(g_{t+1}^*)] + M^2,$$

and the young leave (or would not settle) unless

$$\sum_{i=1}^{3} E_t[\frac{\beta^{i-1}u'(\tilde{c}_{t+1}^i)}{u'(\tilde{c}_t^1)} T_{t+i-1}^i] \leq \sum_{i=1}^{3} E_t[\frac{\beta^{i-1}u'(\tilde{c}_{t+1}^i)}{u'(\tilde{c}_t^1)} v(g_{t+i-1}^*)] + M^1. \qquad (34)$$

All generations favor more borrowing until these constraints start to bind. Because mobility tends to decline as individuals settle down, a reasonable assumption is that $M^1 < M^2 < M^3$. Then the debt-maximizing strategy is to borrow until $T_{t+2}^3 = v(g_{t+2}^*) + M^3$, $T_{t+1}^2 = M^2 - E_{t+1}[\frac{\beta u'(\tilde{c}_{t+2}^3)}{u'(\tilde{c}_{t+1}^2)}]M^3$, and $T_t^1 = M^1 - E_t[\frac{\beta u'(\tilde{c}_{t+1}^2)}{u'(\tilde{c}_t^1)}]M^2$ for all t just meets the budget constraint (22). Debt would be at a natural debt limit.[19]

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[19] This setting does not necessarily imply high debt. If the young are initially unaffiliated, competition across communities would endogenously set M^I at a level that no community could impose taxes on future generations greater than required to finance public service at lowest cost. Then pension underfunding and debt-financed tax deferral are still possible, but on the margin, all tax cuts must be repaid by the same generation.

The policy implications of the general-tax model are somewhat fragile, however, because slight modifications can yield very different conclusions. For example, suppose there is a cross-sectional distribution of moving costs: a fraction $\varphi < 1/2$ of residents can move freely and will do so whenever taxes are greater than in competing communities. Consider again the experiment of a deficit-financed reduction in T_t^1 by followed by a balancing increase in T_{t+1}^2. If mobile agents exit after period t, the utility of residents who stay changes by $\frac{\partial U_t}{\partial (-T_t^1)} = u'(\tilde{c}_t^1)\left(1 - \frac{1}{1-\varphi}\frac{1+\chi_D}{1+\chi_d(d_t^1)}\right)$, which is negative for $\varphi > \frac{\chi_d(d_t^1) - \chi_D}{1+\chi_d(d_t^1)}$. In this case, a majority would vote against a tax cut.

In summary, incentives to incur debt and to underfund pensions are more extreme when individuals are not locked-in by owning property; but debt and unfunded pensions are also more dangerous for individuals who are relatively immobile. Hence the equilibrium policy outcomes are more fragile.

6. Incomplete Markets and Risk Sharing

The main model above assumed complete markets. In reality most retirement savers invest in standard financial instruments such as stocks and bonds. Many DC retirement plans restrict investment options further, sometimes down a small menu of mutual funds. DB plans provide a much more flexible set of benefits, which includes microeconomic insurance features such as annuities and spousal benefits that savers would find costly or impossible to purchase on their own. Managers of DB plans may also have access to a wider range of investment options, through at a cost. This raises questions about robustness of the previous results and it suggests that the complete market model may understate the advantages of DB relative to DC plans. Three issues deserve comment.

(a) Incomplete markets do not overturn the principal arguments against funding. Indeed, if investment options are limited, regulations that impose full funding in the strict sense of "full funding in all states of nature" are even more costly and inefficient because if they require more than 100% funding in all but the "worst" state.

To model incomplete markets, assume only stocks or bonds are traded on liquid financial markets. Stocks are claims on capital and pay a state-contingent return $R^k(s_{t+1})$. Bonds are viewed as safe claims that pay $R^b(s_{t+1} \mid s_t) = \bar{R}^b(s_t) \equiv 1/E_t[m_{t+1}]$ in all states s_{t+1}. In addition, assume Arrow securities are available but only at an intermediation cost $\chi_z \geq 0$. Let $z^+(s_{t+1}) \geq 0$ and $z^-(s_{t+1}) \geq 0$ denote long and short positions in Arrow securities on state s_{t+1}, and assume the period-t prices are $\xi^+(s_{t+1}) = \pi(s_{t+1})m(s_{t+1})(1+\chi_z)$, and $\xi^-(s_{t+1}) = \pi(s_{t+1})m(s_{t+1})/(1+\chi_z)$. The cost $\chi_z \geq 0$ can be interpreted as "markup" over the pricing kernel imposed by insurance companies or investment banks. A setting without Arrow securities is included as limiting case $\chi_z \rightarrow \infty$. (Note that symbols +/- are used differently than in Section 4.) Assume $m(s_{t+1})$ is not spanned by stocks and bonds.

A retirement portfolio can then be written as

$$X(s_{t+1}) = R^k(s_{t+1})x_t^k + R^b(s_{t+1})x_t^b + z^+(s_{t+1}) - z^-(s_{t+1}) \text{ for all } s_{t+1} \tag{35}$$

where x_t^k and x_t^b are holdings of "standard" stocks and bonds (purchased at unit price). The period-t cost of a portfolio of such a portfolio is

$$x_t = x_t^k + x_t^b + \sum_{s_{t+1}} \xi^+(s_{t+1})z^+(s_{t+1}) - \sum_{s_{t+1}} \xi^-(s_{t+1})z^-(s_{t+1}) \tag{36}$$

The cost of a given portfolio is strictly increasing in χ_z unless the $X(s_{t+1})$ lies in the space spanned by $R^k(s_{t+1})$ and $R^b(s_{t+1})$.

Let $U^{DC}(\chi_z, \chi_{DC})$ denote the utility of an individual who invests optimally in a DC pension plan constrained by (35) and (36); U^{DC} is strictly decreasing in χ_z. Similarly, let $U^{DB}(\chi_z, \chi_F; w_t^1, w_{t+1}^2)$ denote an employee's utility working under a DB plan that offers first and second period wages (w_t^1, w_{t+1}^2) and invests optimally. Provided $\chi_F \leq \chi_{DC}$, a DB plan can replicate the optimal DC plan and use the same funding and investment policy. Hence

$$U^{DB}(\chi_z, \chi_F; w_t - x_t^1, ew_{t+1} - x_{t+1}^2) \geq U^{DC}(\chi_z, \chi_{DC}) \tag{37}$$

holds for any χ_z. However, the DB plan is superior in two ways. First, a DB plan has a comparative advantage over DC because it can settle non-traded claims internally—across employee groups or with taxpayer—without incurring transaction cost. The relevance of internal settlement depends in part on risk sharing opportunities (discussed below) and

funding. Second, reduced funding raises taxpayer utility by the same reasoning as in Section 3, so zero funding is still optimal. Zero funding is even more advantageous because it avoids the cost χ_z. If (37) holds with strict inequality, a plan that maximizes η_t^1 or η_t^2 subject to $U^{DB}(\chi_z, \chi_F; w_t^1, w_{t+1}^2) \geq U^{DC}(\chi_z, \chi_{DC})$ will have lower cost (i.e., $w_t^1 < w_t - x_t^1$ and/or $w_{t+1}^2 < ew_{t+1} - x_{t+1}^2$). Reduced cost would justify an expansion of public services and it would increase property values.

One destructive scenario should be noted. Consider a DB plan that has made promises—optimally—that are not fully funded and cannot be backed by traded assets; say, a wage-linked pension. Suppose regulators suddenly demand that the plan be fully funded in all states of nature. This mandate can be satisfied only by buying Arrow securities at marked-up prices or by over-collateralizing the plan with stocks and bonds. Both options are expensive, perhaps more costly than offering a DC plan. Hence uncertainty about funding rules gives plan sponsors incentives to shift (preemptively) from DB plans to DC or to hybrid plans that are more easily backed by standard financial assets.

(b) The optimality of zero contributions by the young should be flagged as a more conditional result. It relies on several implicit assumptions: (i) Debt was assumed state contingent, which allows the young to hedge risks without holding assets. In reality, most borrowers face standard debt contracts with no contingencies except a costly default. As more realistic alternative, consider "inflexible" debt, meaning a debt with exogenous return distribution. It is straightforward to show if debt returns differ sufficiently from the optimal return distribution, the young will make pension contributions—either in a DC plan or via their employer in an optimal DB plan.[20] (ii) Pensions contributions in middle age were assumed unconstrained. If one assumed a binding upper bound on x_{t+1}^2, contributing in young age would be an indirect way to benefit from the tax-boost to returns between periods 2 and

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[20] Note that inflexible debt has ramifications for optimal policy. Because inflexible debt breaks the alignment between pricing kernel and marginal rates of substitution in (20), there is a potential for fiscal policy to improve risk sharing between younger and older generations, as in Bohn (2009). A detailed examination is left for future research because tax instruments are limited in the property tax model and because intergenerational risk sharing is more promising at the national level.

3.[21] However, shifting taxable income from young to old age does not save taxes unless $(1-\tau^3)/(1-\tau^1) > 1$, which may not hold. Moreover, $(1-\tau^3)/(1-\tau^1)$ must be sufficiently greater than one to compensate for the intermediation cost of period-1 debt. (iii) The model abstracts from career employment issue. Unless career employment promises productivity gains, pension benefits for the young are difficult to justify.

(c) Restrictions on the space of financial contracts matter only to the extent that the missing markets are needed to implement the unconstrained optimal allocation. Finance theory implies that the aggregate pricing kernel is the marginal rate of substitution of an investor who bears a representative share of aggregate risks. If output is produced with labor and capital, the fundamental risks are shocks to wages, shocks to the return to capital (including houses). This suggests that m_{t+1} is positively correlated with next period's wage, returns to corporate capital, and house prices.

From budget equation (9), the income of the middle aged is naturally correlated with w_{t+1}. Optimal risk sharing calls for $\tilde{c}_{t+1}^2$ to be perfectly correlated with m_{t+1}. Assuming inflexible debt so the young contribute to pensions, optimal pension accruals from young to middle age should provide equity exposure and hedging against wage risk. From (10), the risk exposure of retirees is largely determined by pensions. Hence optimal pension payments to retirees should provide positive equity exposure and positive exposure to wage risk. (Because both cohorts own houses, optimal hedging against housing risk is unclear.)

These arguments suggest that missing markets for wage-indexed claims are a serious constraint on DC plans.[22] Moreover, because the hedging needs younger and older cohorts go in opposite directions, DB plans can provide internal risk sharing. However, this requires that DB plans promise wage-linked pensions, i.e., pensions that cannot be matched by funding. Final salary plans can be interpreted as practical implementation, because of all possible

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[21] Formally, the shadow value of the upper bound on x_{t+1}^2 would reduce the marginal rate of substitution between middle and old age consumption and enter (17) in a way that reduces $\mu(x_t^1)$, potentially down to zero.

[22] The seriousness of the problem and the ramifications for portfolio choice depend in part on the correlation between wages are equity returns. Baxter-Jermann (1997) and Bohn (1999) estimate a high correlation. Constantinides et al. (2002) argue for a low correlation.

salary indexation schemes, the final salary is closest to the retirement period. Thus the arguments in (a) above apply to wage indexing and final salary plans.

7. Concluding comments

This paper examines state and local pension funding in an overlapping generations model with intermediation cost. I find that full funding is difficult to justify when taxpayers face a cost of borrowing and they enjoy tax advantages in saving for retirement. The resulting distortions raise the opportunity cost of funds above the rates of return available on fund investment. Put simply: Why should people pay taxes to fund a pension plan that buys Treasury bonds yielding 4% when they are paying 15% interest on their credit cards and 7% on car loans? Though underfunding sounds ominous and there may be good reasons for balanced budgets, one must recognize that funding is costly. These arguments imply that zero funding would be optimal.

A plausible countervailing force is legal ambiguity, the risk that unfunded pension promises may not be enforceable. Under some conditions, funding can serve collateral, but except under extreme assumptions, it does not justify 100% funding. Excessive funding requirements raise the cost of DB relative to DC plans. The trend in U.S. accounting rules towards higher and higher funding requirements has largely killed private sector DB plans. One must wonder if public sector DB plans await the same fate. This would be unfortunate because DB pensions have important efficiency advantages over defined contribution plans.

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Table 1: What Percentages of U.S. Families are Borrowers?

Generation (in Model)	Age Bracket	Percentage of Families who hold				
		Any Debt	Type of Debt:			Memo: Retirement Account
			Mortgage	Installment Loan	Credit card	
		(1)	(2)	(3)	(4)	(5)
Young	<35	**83.5%**	*37.3%*	**65.2%**	*48.5%*	*41.6%*
	35-44	**86.2%**	**59.5%**	**56.2%**	**51.7%**	**57.5%**
Middle	45-54	**86.8%**	**65.5%**	**51.9%**	**53.6%**	**64.7%**
	55-64	**81.8%**	**55.3%**	*44.6%*	*49.9%*	**60.9%**
Old	65-74	**65.5%**	*42.9%*	*26.1%*	*37.0%*	**51.7%**
	≥75	*31.4%*	*13.9%*	*7.0%*	*18.8%*	*30.0%*

Memo: Interest Rate Spreads over Treasuries

		Prime vs. TB 3mo	Fixed 30y vs Tr.10yr	Car loan vs Tr.3yr	Av. Card vs TB 3mo
Current (July 2010)		3.1%	1.7%	4.9%	14.2%
Average 1990-2009		3.2%	1.7%	NA	NA

Legend: **Bold** = majority. *Italics* = minority.

Sources: Percentage of families' data from Survey of Consumer Finances 2007. Mortgage refers to mortgages secured by primary residence. Rate spread averages are own calculations based on FRB release H.15. Current spreads from the Wall Street Journal for July 9, 2010. Prime vs. TB 3mo = Prime rate minus 3-month Treasury bill rate. Fixed 30y vs Tr.10yr = 30-year fixed rate mortgage rate minus 10-year Treasury rate. Car loan vs Tr.3yr = Average rate on 36 month car loans minus 3-year Treasury rate (Per SCF, car loans are the most common installment loans). Av. Card vs TB 3mo = Average rate on credit cards minus 3-month Treasury bill rate.

Table 2: Funding as collateral against legal ambiguity or default

(a) Maximum probability of enforcement problems so ZERO funding is optimal

Funding cost		Annual	0.25%	0.5%	1.0%
Borrowing cost		Annual	0.75%	1.5%	3.0%
Cost factor		20-year	**0.224**	**0.505**	**1.316**
Pension Dependence	Risk aversion	**Risk factor $\Delta u'$**	(1)	(2)	(3)
25%	2	**0.8**	28.8%	64.9%	(never)
25%	3	**1.4**	16.3%	36.8%	96.0%
25%	4	**2.2**	10.4%	23.4%	60.9%
50%	2	**3.0**	7.5%	16.8%	43.9%
50%	3	**7.0**	3.2%	7.2%	18.8%
50%	4	**15.0**	1.5%	3.4%	8.8%
75%	2	**15.0**	1.5%	3.4%	8.8%
75%	3	**63.0**	0.4%	0.8%	2.1%
75%	4	**255.0**	0.1%	0.2%	0.5%

(b) Optimal funding ratios for given probabilities of enforcement problems

Default Probability:		**5%**	**10%**	**25%**	**25%**
Funding cost		0.25%	0.25%	0.25%	*0.50%*
Borrowing cost		0.75%	0.75%	0.75%	*1.50%*
Optimal $\Delta u'$ = Cost/Default		4.477	2.239	0.895	2.019
Pension Dependence	**Risk aversion**	(1)	(2)	(3)	Case of higher cost
25%	**2**	0	0	0	0
25%	**4**	0	0	41%	*3%*
25%	**10**	37%	56%	75%	*58%*
50%	**2**	0	11%	45%	*15%*
50%	**4**	31%	49%	70%	*52%*
50%	**10**	69%	78%	88%	*79%*
75%	**2**	24%	41%	64%	*43%*
75%	**4**	54%	66%	80%	*68%*
75%	**10**	79%	85%	92%	*86%*