Targeting Long Rates in a Model with Segmented Markets

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January 17, 2014

Abstract: This paper develops a model of segmented financial markets in which the net worth of financial institutions limits the degree of arbitrage across the term structure. The model is embedded into the canonical Dynamic New Keynesian (DNK) framework. Our principle results include the following. First, there are welfare gains to having the central bank respond to the term premium, eg., including the term premium in the Taylor Rule. But the sign of the preferred response depends upon the type of shocks driving the business cycle. Second, a policy that directly targets the term premium sterilizes the real economy from shocks originating in the financial sector.

The views expressed in this paper are those of the authors, and not necessarily those of the Federal Reserve Bank of Cleveland, or of the Board of Governors of the Federal Reserve System or its staff. We have received excellent research assistance from Dasha Safonova.

1. Introduction.

In the aftermath of the 2008 financial crisis many central banks have adopted unconventional policies, including outright purchases of long term government debt. These bond purchases raise a number of research questions for macro theory. Under what conditions can such purchases have aggregate effects? If they have aggregate consequences, how do term premia movements affect inflation and economic activity? What are appropriate policy rules for such interventions? To answer such questions, this paper develops a model of the term premium in which central bank purchases can affect the yield structure independently of the anticipated path of short term interest rates. The model is embedded into an otherwise canonical medium-scale DNK model where long-term bonds are necessary to finance investment purchases. This implies that both new and old policy questions can be examined in a unified framework.

The key features of the model include the following. First, the short term bond market is segmented from the long term bond market in that only financial intermediaries can purchase long term debt. Households can access the long-term debt instruments indirectly by providing deposits to intermediaries. Second, the ability of intermediaries to arbitrage the yield gap between the short term deposit rate and long term lending rate is limited by net worth. That is, a simple hold-up problem constrains the amount of deposits that can be supported by a given level of intermediary net worth. Third, the intermediary faces adjustment costs in rapidly varying the size of its portfolio in the wake of shocks. These assumptions imply that central bank purchases of long-term bonds will have a significant effect on long yields. Finally these long-term yields affect real economic activity because of our final assumption: capital investment is financed by the issuance of long term bonds which sell in the same market that absorbs long term Treasuries. Taken together, these assumptions imply that central bank purchases of long-term bonds will have a significant and persistent effect on long yields and real activity.

We use the model to consider the efficacy of alternative policies linked to the term premium. This is a natural policy in the context of the model as the distortion arising from market segmentation is, to a linear approximation, equal to the term premium. Hence, we show that there are significant welfare gains to including the term premium in a traditional Taylor rule operating on the short term rate. We also consider policies that utilize a Taylor-type rule over the long rate. Such a long rate policy sterilizes the rest of the model economy from shocks originating in the financial system. This sterilization is directly analogous to the classic Poole (1970) result that a FFR targets sterilize the economy from money demand shocks.

The papers closest in spirit to the current work are Gertler and Karadi (2011, 2013) and Chen, Curdia, and Ferrero (2013). There are two crucial similarities between these papers and the present work. First, there is some friction that limits the ability to arbitrage across the short-term and long-term bond markets. This implies that the long rate is not the expected average of short rates, i.e., there is a term premium. Second, the market segmentation has real effects because some portion of real activity is financed in the segmented market. Gertler and Karadi (2013) assume that the entire capital stock is re-financed each period by the purchase of equity claims in this market by intermediaries. Chen et al. (2013) assume that a small subset of consumers finance their consumption in the segmented market. In contrast, the current paper assumes that new investment is financed in the segmented market with the issuance of long term debt. Both of these assumptions will magnify the effects of segmentation because investment is the most interest-sensitive component of aggregate expenditure, and the long term debt assumption implies that persistent interest rate movements have larger effects. Hence, a central bank bond purchase policy will have a much larger effect in the present paper than in the models of Gertler and Karadi (2013) and Chen et al. (2011).

The paper proceeds as follows. The next section develops the theoretical model, culminating in a discussion of calibration. Section 3 presents our quantitative results including how the segmentation affects the IRFs to shocks, and the efficacy of central bank policies that directly or indirectly target the term premium. Section 4 concludes.

2. The Model.

The economy consists of households, financial intermediaries (FI's), and firms. We discuss each in turn.

<u>Households.</u>

Households are infinitely lived with preferences over consumption (C_t) and labor (L_t) given by:

$$E_0 \sum_{j=0}^{\infty} \beta^j \left\{ ln(C_{t+j} - hC_{t+j-1}) - B \frac{L_{t+j}^{1+\eta}}{1+\eta} \right\} \tag{1}$$

The household earns income by selling its labor services and renting capital to the intermediate goods firm. The household has two means of intertemporal smoothing: short term deposits (D_t) in the financial intermediaries (FI), and accumulation of physical capital (K_t). Households also have access to the market in short term government bonds ("T-bills"). But since T-bills are perfect substitutes with deposits, and the supply of T-bills moves endogenously to hit the central bank's short-term interest rate target, we treat D_t as the household's net resource flow into the FI's. To introduce a need for intermediation, we assume that all investment purchases must be financed by issuing new "investment bonds" that are ultimately purchased by the FI. We find it convenient to use the perpetual bonds suggested by Woodford (2001). In particular, these bonds are perpetuities with cash flows of 1, κ_I, κ_I^2, etc. Let Q_t^I denote the time-t price of a new issue. Given the time pattern of the perpetuity payment, the new issue price Q_t^I summarizes the prices at all maturities, eg., $\kappa_I Q_t^I$ is the time-t price of the perpetuity issued in period t-1. The duration and (gross) yield to maturity on these bonds are defined as: duration = $(1-\kappa_I)^{-1}$, gross yield to maturity = $Q_t^{-1} + \kappa_I$. Let CI_t denote the number of new perpetuities issued in time-t to finance investment. In time-t, the household's nominal liability on past issues is given by:

$$F_{t-1} = CI_{t-1} + \kappa_I CI_{t-2} + \kappa_I^2 CI_{t-3} + \cdots \tag{2}$$

We can use this recursion to write the new issue as

$$CI_t = (F_t - \kappa_I F_{t-1}) \tag{3}$$

The household constraints are thus given by:

$$C_t + \frac{D_t}{P_t} + P_t^k I_t + \frac{F_{t-1}}{P_t} \leq W_t L_t + R_t^k K_t - T_t + \frac{D_{t-1}}{P_t} R_{t-1} + \frac{Q_t^I (F_t - \kappa_I F_{t-1})}{P_t} + div_t \tag{4}$$

$$K_{t+1} \leq (1-\delta) K_t + I_t \tag{5}$$

$$P_t^k I_t \leq \frac{Q_t^I (F_t - \kappa_I F_{t-1})}{P_t} = \frac{Q_t^I CI_t}{P_t} \tag{6}$$

where P_t is the price level, P_t^k is the real price of capital, R_{t-1} is the gross nominal interest rate on deposits, W_t is the real wage, R_t^k is the real rental rate, T_t are lump-sum taxes, and div_t denotes the dividend flow from the FI's. The household also receives a profit flow from the intermediate goods producers and the new capital producers, but this is entirely standard so we dispense from this added notation for simplicity. The “loan-in-advance” constraint (6) will increase the private cost of purchasing investment goods. Although for simplicity we place capital accumulation within the household problem, this model formulation is isomorphic to an environment in which household-owned firms accumulate capital subject to the loan constraint. In any event, the first order conditions to the household problem include:

$$\Lambda_t = E_t \beta \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \tag{7}$$

$$BL_t^{\eta} = W_t \Lambda_t \tag{8}$$

$$\Lambda_t P_t^k M_t = E_t \beta \Lambda_{t+1} [R_t^k + (1-\delta) P_{t+1}^k M_{t+1}] \tag{9}$$

$$\Lambda_t Q_t^I M_t = E_t \beta \Lambda_{t+1} \frac{[1 + \kappa_I Q_{t+1}^I M_{t+1}]}{\Pi_{t+1}} \tag{10}$$

where Λ_t denotes the marginal-utility of time-t consumption, and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is gross inflation. Expressions (7) and (8) are the familiar Fisher equation and labor supply curve. The capital accumulation expression (9) is distorted relative to the familiar by the time-varying distortion M_t, where $M_t \equiv 1 + \frac{\vartheta_t}{\Lambda_t}$, and ϑ_t is the multiplier on the loan-in-advance constraint (6). The endogenous behavior of this distortion is fundamental to the real effects arising from market segmentation.

Financial Intermediaries.

The FI’s in the model are a stand-in for the entire financial nexus that uses accumulated net worth (N_t) and short term liabilities (D_t) to finance investment bonds (F_t) and the long-term government bonds (B_t). The FIs are the sole buyers of the investment bonds and long term government bonds. We again assume that government debt takes the form of Woodford-type perpetuities that provide payments of 1, κ, κ^2, etc. Let Q_t denote the price of a new-debt issue at time-t. The time-t asset value of the current and past issues of investment bonds is:

$$Q_t^I CI_t + \kappa_I Q_t^I [CI_{t-1} + \kappa_I CI_{t-2} + \kappa_I^2 CI_{t-3} + \cdots] = Q_t^I F_t \tag{11}$$

The FIs balance sheet is thus given by:

$$\frac{B_t}{P_t}Q_t + \frac{F_t}{P_t}Q_t^I = \frac{D_t}{P_t} + N_t \tag{12}$$

Note that on the asset side, investment lending and long term bond purchases are perfect substitutes to the FI. Let $R_t^L \equiv E_t\left(\frac{1+\kappa Q_{t+1}}{Q_t}\right) = E_t\left(\frac{1+\kappa_I Q_{t+1}^I}{Q_t^I}\right)$, denote the common one-period gross return on these assets. The financial friction arises on the other side of the balance sheet: FI's ability to attract deposits will be limited by their net worth. We will use a simple hold-up problem to generate this constraint, but a wide variety of informational restrictions will generate the same constraint. Let X_t denote the bank's real asset portfolio:

$$X_t \equiv \frac{B_t}{P_t}Q_t + \frac{F_t}{P_t}Q_t^I. \tag{13}$$

This portfolio has expected return R_t^L during time t. At the beginning of period t+1, but before aggregate shocks are realized, the FI can choose to default on its planned repayment to depositors. In this event, depositors can seize at most fraction Φ_t of the FI's assets. If the FI chooses to repay depositors, the FI is left with $(R_t^L X_t - R_t \frac{D_t}{P_t})$. If the FI defaults, the FI is left with $(1-\Phi_t)R_t^L X_t$, but is otherwise free to continue functioning in subsequent periods.[1] To ensure that the FI will always re-pay the depositor, the time-t hold-up constraint is thus given by:

$$R_t \frac{D_t}{P_t} \leq \Phi_t R_t^L X_t \tag{14}$$

Using the balance sheet identity and re-arranging we have:

$$X_t \leq N_t L\left(\frac{R_t^L}{R_t}\right) \tag{15}$$

where the leverage function is given by:

$$L\left(\frac{R_t^L}{R_t}\right) \equiv \left[1 - \Phi_t \frac{R_t^L}{R_t}\right]^{-1} \tag{16}$$

Log-linearizing this expression we have:

$$(r_t^L - r_t) = \nu l_t - \phi_t \tag{17}$$

[1] This is in contrast to Gertler and Karadi (2011, 2013) who assume that the bank exits the industry.

where $\nu \equiv (L_{ss} - 1)^{-1}$, is the elasticity of the interest rate spread to leverage, and $\phi_t \equiv ln(\Phi_t)$, follows an AR(1) process:

$$\phi_t = (1 - \rho_\phi)\phi_{ss} + \rho_\phi \phi_{t-1} + \varepsilon_{\phi,t}, \tag{18}$$

Decreases in ϕ_t will exacerbate the hold-up problem, and thus are "credit shocks" which will increase the spread and lower real activity.

Qualitatively the log-linearized expression (17) for leverage is identical to the corresponding relationship in the more complex costly-state- verification (CSV) environment of, for example, Bernanke, Gertler, and Gilchrist (1999). In a CSV model, the primitives include: (i) idiosyncratic risk, (ii) death rate, and (iii) monitoring cost. One typically chooses these to match values for (i) leverage, (ii) interest rate spread, and (iii) default rate. The hold-up model has only two primitives: (i) the impatience rate ζ, and (ii) the fraction of assets that can be seized Φ. In comparison to the hold-up model, the extra primitive in the CSV framework thus allows it to match one more moment of the financial data (default rates). One important quantitative difference is that interest rate spreads are more responsive to leverage in our framework than in the CSV model calibrated to the same steady state leverage. For example, suppose we calibrated a CSV model to a leverage of 6.0, a risk premium of 100 bp, and a quarterly default rate of 0.205% (the default rate in the hold-up model is 0%). This would imply ν = 0.097. In the hold-up model analyzed here, a leverage of 6.0 implies ν = 0.20, about twice as large as the CSV counterpart. This is part of the reason why financial frictions have comparably larger real effects in our model.

The hold-up problem would not remain a constraint if the FI could accumulate sufficient net worth. At the beginning of period t, the FI has profits on its portfolio equal to

$$prof_t \equiv \left[\frac{Q_{t-1}B_{t-1}}{P_t}\left(\frac{1+\kappa Q_t}{Q_{t-1}}\right) + \frac{Q_{t-1}^I F_{t-1}}{P_t}\left(\frac{1+\kappa_I Q_t^I}{Q_{t-1}^I}\right) - R_{t-1}\frac{D_{t-1}}{P_t}\right] \tag{19}$$

The FI will pay out some of these as dividends to the household, and retain the rest as net worth for subsequent activity. In making this choice, the FI discounts dividend flows using the household's pricing kernel, augmented with additional impatience. The FI's decision problem is given by:

$$max\ E_0 \sum_{j=0}^{\infty} (\beta\zeta)^j \Lambda_{t+j} div_{t+j} \tag{20}$$

subject to

$$div_t + N_t[1 + N_t f(N_t)] = prof_t \tag{21}$$

where $f(N_t) \equiv \frac{\psi_n}{2}\left(\frac{N_t - N_{ss}}{N_{ss}}\right)^2$. The FI's net worth decision is given by:

$$\Lambda_t[1 + N_t f'(N_t) + f(N_t)] = E_t \beta\zeta\Lambda_{t+1} \frac{R_t^L}{\Pi_{t+1}} \tag{22}$$

Equations (15) and (22) are fundamental to the model as they summarize the limits to arbitrage between the return on long term bonds and the rate paid on short term deposits. The net worth constraint (15) limits the FI's ability to attract deposits and eliminate the arbitrage opportunity between the deposit and lending rate. Hence, the expectations theory of the term structure holds within the long bond market, but not between the short and long debt market. In essence, the segmentation decouples the short rate from the rest of the term structure. Increases in net worth allow for greater arbitrage and thus can eliminate this market segmentation. Equation (22) limits this arbitrage in the steady-state ($\zeta < 1$) and dynamically ($\psi_n > 0$).[2] Since the FI is the sole means of investment finance, this market segmentation means that central bank purchases that alter the supply of long term debt will have repercussions for investment loans because net worth and deposits cannot quickly sterilize the purchases.

Final good producers.

Perfectly competitive firms produce the final consumption good Y_t combining a continuum of intermediate goods according to the CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1/(1+\epsilon_p)}\, di\right]^{1+\epsilon_p} \tag{23}$$

[2] In particular, comparing equations (7) and (22) we see that without adjustment costs the spread between the loan rate and the return on short-term T-bills would be constant over time.

Profit maximization and the zero profit condition imply that the price of the final good, P_t, is the familiar CES aggregate of the prices of the intermediate goods.

Intermediate goods producers.

A monopolist produces the intermediate good *i* according to the production function

$$Y_t(i) = A_t K_t(i)^{\alpha} L_t(i)^{1-\alpha} \tag{24}$$

where $K_t(i)$ and $L_t(i)$ denote the amounts of capital and labor employed by firm *i*. The variable lnA_t is the exogenous level of TFP and evolves according to:

$$lnA_t = \rho_A lnA_{t-1} + \varepsilon_{a,t}, \tag{25}$$

Every period a fraction θ_p of intermediate firms cannot choose its price optimally, but resets it according to the indexation rule

$$P_t(i) = P_{t-1}(i)\Pi_{t-1}^{\iota_p}, \tag{26}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation. The remaining fraction of firms chooses its price $P_t\,(i)$ optimally, by maximizing the present discounted value of future profits

$$E_t\left\{\sum_{s=0}^{\infty} \theta_p^s \, \frac{\beta^s \Lambda_{t+s}/P_{t+s}}{\Lambda_t/P_t}\left[P_t(i)\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)Y_{t+s}\,(i) - W_{t+s}L_{t+s}(i) - P_{t+s}\rho_{t+s}K_{t+s}(i)\right]\right\} \tag{27}$$

where the demand function comes from the final goods producers.

New Capital Producers.

New capital is produced according to the production technology that takes $\mathrm{I_t}$ investment goods and transforms them into $\mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t$ new capital goods. The time-t profit flow is thus given by

$$P_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - I_t \tag{28}$$

where the function *S* captures the presence of adjustment costs in investment, as in Christiano et al. (2005), and is given by:

$$S\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\psi_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

These firms are owned by households and discount future cash flows with Λ_t. The investment shock follows the stochastic process

$$log\mu_t = \rho_\mu log\mu_{t-1} + \varepsilon_{\mu,t}, \tag{29}$$

where $\varepsilon_{\mu,t}$ is *i.i.d.* N $\left(0, \sigma_\mu^2\right)$. Following Justiniano, Primiceri, and Tambalotti (2010,2011), we call these MEI shocks, for "marginal efficiency of investment." The chief source of the business cycle in the model will be TFP and MEI shocks.

Central Bank Policy.

We assume that the central bank follows a familiar Taylor rule over the short rate (T-bills and deposits):

$$ln(R_t) = (1-\rho)\, ln(R_{ss}) + \rho\, ln(R_{t-1}) + (1-\rho)(\tau_\pi \pi_t + \tau_y y_t^{gap}) \tag{30}$$

where $y_t^{gap} \equiv (Y_t - Y_t^f)/Y_t^f$, denotes the deviation of output from its flexible price counterpart. We will think of this as the Federal Funds Rate (FFR). Below we will also investigate the efficacy of putting the term-premium into the Taylor rule. The supply of T-bills is endogenous, varying as needed to support the FFR target. As for the long term policy, the central bank will choose between: (i) an exogenous path for the quantity of long term debt available to FIs, or (ii) a policy rule for the long term bond yield. We will return to this below.

Fiscal policy is entirely passive. Government expenditures are set to zero. Lump sum taxes move endogenously to support the interest payments on the short and long debt.

Loglinearized Model.

To gain further intuition and to derive the term premium, we first log-linearize the model. Let $b_t \equiv \ln\left(\frac{\bar{B}_t}{\bar{B}_{ss}}\right)$, and $f_t \equiv \ln\left(\frac{\bar{F}_t}{\bar{F}_{ss}}\right)$, where $\bar{B}_t \equiv Q_t \frac{B_t}{P_t}$, and $\bar{F}_t \equiv Q_t^I \frac{F_t}{P_t}$, denote the real market value of the bonds available to FIs. We will focus on bonds of 10-year maturities, so R_t^{10} will denote their gross yield. Using lower case letters to denote log deviations, the log-linearized model is given by the following:

$$\lambda_t = \frac{1}{(1-\beta h)(1-h)} E_t[\beta h c_{t+1} - (1+\beta h^2)c_t + h c_{t-1}] \tag{32}$$

$$\eta L_t = w_t + \lambda_t \tag{33}$$

$$\lambda_t = \lambda_{t+1} + r_t - \pi_{t+1} \tag{34}$$

$$\lambda_t + p_t^k + m_t = E_t\{\lambda_{t+1} + [1-\beta(1-\delta)]r_{t+1}^k + \beta(1-\delta)(p_{t+1}^k + m_{t+1})\} \tag{35}$$

$$m_t + r_t + q_t^i = \beta\kappa_I E_t(q_{t+1}^i + m_{t+1}) \tag{36}$$

$$(1-\kappa_I)(p_t^k + i_t) = f_t - \kappa_I(f_{t-1} + q_t^i - q_{t-1}^i - \pi_t) \tag{37}$$

$$r_t^L = E_t \frac{\kappa_I q_{t+1}^i}{R_{ss}^L} - q_t^i \tag{38}$$

$$r_t^L = E_t \frac{\kappa q_{t+1}}{R_{ss}^L} - q_t \tag{39}$$

$$r_t^{10} = -\left(\frac{R_{ss}^L - \kappa}{R_{ss}^L}\right) q_t \tag{40}$$

$$(r_t^L - r_t) = \left(\frac{1}{L_{ss}-1}\right)\left[\frac{\bar{B}_{ss}}{L_{ss}N_{ss}} b_t + \left(1 - \frac{\bar{B}_{ss}}{L_{ss}N_{ss}}\right) f_t - n_t\right] - \phi_t \tag{41}$$

$$\psi_n n_t = (r_t^L - r_t) \tag{42}$$

$$\frac{\bar{B}_{ss}}{N_{ss}} b_t + \left(L_{ss} - \frac{\bar{B}_{ss}}{N_{ss}}\right) f_t = n_t + (L_{ss}-1)d_t \tag{43}$$

$$w_t = mc_t + mpl_t \tag{44}$$

$$r_t^k = mc_t + mpk_t \tag{45}$$

$$\pi_t = \frac{\kappa_\pi}{1+\beta\iota_p} mc_t + \frac{\beta}{1+\beta\iota_p} E_t\pi_{t+1} + \frac{\iota}{1+\beta\iota_p}\pi_{t-1} \tag{46}$$

$$p_t^k = \psi_i[(i_t - i_{t-1}) - \beta E_t(i_{t+1} - i_t)] - \mu_t \tag{47}$$

$$\left(1-\frac{I_{ss}}{Y_{ss}}\right)c_t+\frac{I_{ss}}{Y_{ss}}i_t=a_t+\alpha k_t+(1-\alpha)L_t \tag{48}$$

$$k_{t+1}=(1-\delta)k_t+\delta(\mu_t+i_t) \tag{49}$$

$$r_t=\rho r_{t-1}+(1-\rho)(\tau_\pi\pi_t+\tau_y y_t^{gap}) \tag{50}$$

To close the model, we need one more equation outlining the policy rule for the long term debt market. Before a discussion of these policy options, several comments are in order.

First, equation (35) highlights the economic distortion, m_t, arising from the segmented markets. Solving this forward we have:

$$p_t^k+m_t=\sum_{j=0}^{\infty}[\beta(1-\delta)]^j\left\{[1-\beta(1-\delta)]r_{t+j}^k-(r_{t+j}-\pi_{t+j+1})\right\} \tag{51}$$

As is clear from (51), the segmentation distortion, m_t, acts like a mark-up or excise tax on the price of new capital goods. What is this distortion? Using (36) and (38) we have

$$m_t=\sum_{j=0}^{\infty}(\beta\kappa_I)^j\Xi_{t+j}, \tag{51}$$

where

$$\Xi_{t+j}\equiv\beta\kappa_I q_{t+j+1}^i-q_{t+j}^i-r_{t+j}\approx r_{t+j}^L-r_{t+j} \tag{52}$$

The distortion is thus the discounted sum of the future one-period loan to deposit spreads. As discussed above, this spread exists because of the assumed market segmentation.

Second, the *term premium* can be defined as the difference between the observed yield on a 10-year bond (see (40)) and the corresponding yield implied by applying the expectation hypothesis (EH) of the term structure to the series of short rates. The price of this hypothetical EH bond satisfies

$$r_t=E_t\frac{\kappa q_{t+1}^{EH}}{R^L}-q_t^{EH} \tag{53}$$

while its yield is given by

$$r_t^{EH,10}=\left(\frac{R_{ss}-\kappa}{R_{ss}}\right)q_t^{EH}. \tag{54}$$

Using these definitions, the term premium can be expressed as

$$term\ premium\equiv tp_t\equiv\left(r_t^{10}-r_t^{EH,10}\right)=-\left(\frac{R_{ss}^L-\kappa}{R_{ss}^L}\right)q_t+\left(\frac{R_{ss}-\kappa}{R_{ss}}\right)q_t^{EH} \tag{55}$$

Solving the bond prices in terms of the future short rates, we have

$$tp_t = \left(\frac{R_{ss}^L - \kappa}{R_{ss}^L}\right) \sum_{j=0}^{\infty} \left(\frac{\kappa}{R_{ss}^L}\right)^j r_{t+j}^L - \left(\frac{R_{ss} - \kappa}{R_{ss}}\right) \sum_{j=0}^{\infty} \left(\frac{\kappa}{R_{ss}}\right)^j r_{t+j}$$

$$\approx (1 - \beta\kappa) \sum_{j=0}^{\infty} (\beta\kappa)^j \left(r_{t+j}^L - r_{t+j}\right) \tag{56}$$

Comparing (51) and (56), the distortion m_t is closely proxied by the term premium. One minor difference is that the weights in the term premium are linked to the duration of the government bond via κ, while the segmentation distortion (51) is linked the duration of the investment bond (κ_I). In any event, a policy that eliminates fluctuations in the term premium will largely eliminate fluctuations in the market segmentation distortion.

Third, the loan-deposit spread arises because of the segmentation effects summarized in (41)-(42). Equation (41) expresses the endogenous response of leverage to higher expected returns on intermediation, while equation (42) summarizes the FIs desire to accumulate more net worth in response to the profit opportunity of the spread. The model's dynamics collapse to the familiar DNK model if we set $\psi_n = 0$, so that net worth can move instantaneously to eliminate all arbitrage opportunities. But if $\psi_n > 0$, then the segmentation acts like an endogenous adjustment cost to investment. That is, increases in investment necessitate an increase in investment bonds (37), but this drives up the one-period spread (41) and thus m_t. The net worth adjustment cost (42) implies that this effect cannot be entirely undone by movements in net worth.

Fourth, the previous suggests that a policy that stabilizes the term premium will likely be welfare improving (unless the interaction with the sticky price distortion is significant). This suggests the efficacy of a central bank including the term premium in a Taylor type rule. But we can take this argument one step further. Under a policy that directly targets the term premium the supply of long debt held by FIs will be endogenous. In particular, (41) and (43) separate out from the rest of the model, and define the behavior of long bonds and FI deposits that move endogenously to support the long rate target. This implies that "credit shocks", those proxied by ϕ_t in (41), will have no effect on real activity or inflation. That is, a long rate policy sterilizes the real economy from financial shocks. This is analogous to the classic result of Poole (1970) in which an interest rate target sterilizes the real economy from shocks to money demand.

Fifth and finally, the assumption that the long bonds are nominal implies that monetary policy shocks will have real effects even in a flexible price model. This is seen most clearly in (37). Innovations in inflation will erode the existing real value of investment debt thus making increased issuance less costly. This effect disappears if the debt is only one period ($\kappa_I = 0$), or if the debt is indexed to inflation (κ_I is a real payment).

Debt market policies.

To close the model, we need one more restriction that will pin down the behavior in the long debt market. We will consider two different policy regimes for this market: (i) exogenous debt, and (ii) endogenous debt. We will discuss each in turn.

Exogenous debt. The variable b_t denotes the real value of long term government debt on the balance sheet of FI's. There are two distinct reasons why this variable could fluctuate. First, the central bank could engage in long bond purchases ("quantitative easing," or QE). Second, the fiscal authority could alter the mix of short debt to long debt in its maturity structure. We will model both of these scenarios as exogenous movements in long debt. Under either scenario, the long yield r_t^{10} will be endogenous. To model a persistent and hump-shaped QE policy shock we will use an AR(2):

$$b_t = \rho_1^b b_{t-1} + \rho_2^b b_{t-2} + \epsilon_t^b \tag{57}$$

Within such an exogenous debt regime, we will also consider policies in which the Taylor rule for the short rate responds to some measure of the term premium:

$$r_t = \rho r_{t-1} + (1-\rho)(\tau_\pi \pi_t + \tau_y y_t^{gap} + \tau_{tp} tp_t) \tag{58}$$

where the term premium (tp_t) is defined as in (56). As noted earlier, there are reasons to think that such a policy may be welfare-improving.

Endogenous debt. The polar opposite scenario is a policy under which the central bank targets the term-premium tp_t, in a fashion similar to the Taylor rule for the short rate:

$$tp_t = \rho_{10} tp_{t-1} + (1-\rho_{10})(\tau_\pi^{10} \pi_t + \tau_y^{10} y_t). \tag{59}$$

Under this policy regime the level of long debt b_t will be endogenous. We will focus on policies that peg the term premium at steady state, ie., $tp_t = 0$. When the central bank pegs the term premium at steady state it effectively becomes the marginal lender to the private sector for investment. To see this, note from (56) that one way to achieve a term premium peg is to hold constant the spread of the period return on investment bonds over the deposit rate at all times. But then from the balance sheet of the intermediary and the leverage constraint, FI net worth and household deposits are constant. Hence, any increase of FI holdings of investment debt is achieved via the central bank purchasing government bonds. The proceeds from this sale effectively finances loans for investment.

Calibration.

Much of the calibration is standard and is similar to Gertler and Karadi (2013): $\beta = 0.99$, $h = 0.8$, $\eta = 0.25$, $\iota_p = 1$, $\epsilon_p = 5$, $\theta_p = 0.85$, $\psi_i = 2$. Monetary policy over the funds rate is given by $\rho = 0.8$, $\tau_\pi = 1.5$, and $\tau_y^{gap} = 0.5$. The atypical parameters for calibration are those surrounding the FI. We will use evidence on interest rate spreads and leverage to pin down two primitive parameters. The steady-state loan-deposit spread and leverage ratio are given by:

$$\zeta = \left(\frac{R_{ss}^L}{R_{ss}}\right)^{-1}$$

$$L_{ss} = \left[1 - \Phi_{ss}\left(\frac{R_{ss}^L}{R_{ss}}\right)\right]^{-1}$$

We will choose the parameters ζ and ψ, to match an interest rate spread of 100 annual bp, and a leverage level of 6. This is the same calibration as in Gertler and Karadi (2013). The government and investment bonds will both be calibrated to a duration of 40 quarters, $(1-\kappa_I)^{-1} = (1-\kappa)^{-1} = 40$. We also need to calibrate the balance sheet proportion, $\frac{\bar{B}_{ss}}{N_{ss}}$. This is proportional to the fraction of FI assets held as long term debt:

$$\frac{\bar{B}_{ss}}{N_{ss}} = \frac{\bar{B}_{ss}}{\bar{F}_{ss}+\bar{B}_{ss}} * L_{ss}$$

Consistent with studies of bank balance sheets, we set the ratio of government securities to total bank assets to $\frac{\bar{B}_{ss}}{\bar{F}_{ss}+\bar{B}_{ss}} = 40\%$.

Finally, the adjustment cost parameter ψ_n drives the link between net worth accumulation and the loan-deposit spread. We will choose this parameter to be consistent with the empirical evidence on the effect of the Fed's QE policies on the 10 year bond rate. We will later provide sensitivity analysis showing that only a modest degree of adjustment costs are necessary to produce a significant change in the term premium and real economic activity.

Figure 1 graphs the change in the Fed's bond portfolio relative to the government debt in the hands of the domestic public. The QE policies are quite apparent. We will consider a QE shock that decreases b_t by 6.5%, comparable to the magnitude in Figure 1 (roughly $300 billion).[3] To match the persistent nature of this expansion we set $\rho_1^b = 1.8$, and $\rho_2^b = -0.81$. Empirical estimates of the response of the 10 year yield to these QE shocks vary from no effect to over 45 bp (eg., the evidence discussed in Chen et al. (2013)). We set $\psi_n = 1$, implying that the long term yield moves by 23 bp in response to our QE shock, a response in the middle of the estimates in the literature (about 7 bp for each $100 billion purchase).

3. Quantitative Results.

a. <u>QE shocks.</u>

The impulse response to the QE shock is exhibited in Figure 2. The policy shock has a persistent effect on the 10 year yield, with all of this initial movement being driven by changes in the term premium. This term premium effect dissipates as net worth responds and segmentation returns to steady state levels, so that the long rate is eventually driven by the path of the short rate. The policy has a persistent and significant effect on investment and output, while consumption is little changed for the first 10 quarters. The demand component of

[3] Recall that b_t is the amount of government debt held by the FI's, so that a QE shock is a decrease in b_t.

the shock naturally leads to an increase in inflation, and thus a policy-induced increase in the funds rate. The funds rate eventually overshoots its long run level, thus leading to a persistent decline in the long rate. The real-effects of the QE shock is larger and more persistent than in other models of QE for two reasons. First, the long term nature of the investment debt (10 year maturity) amplifies the effect of a given movement in the yield. And second, the FIs finance investment purchases, the most elastic portion of aggregate output. In contrast, Gertler and Karadi (2011, 2013) have the entire capital stock financed by the FIs, while Chen et al. (2011) have consumption of one class of agents linked to the segmented market.

Figure 2 also contains the effect of pegging the funds rate at steady-state for four periods in the wake of the QE shock. This is meant to mimic a zero-lower-bound (ZLB) experiment. Evidently the ZLB becomes a binding constraint in the data because of other shocks to which the central bank is responding. But our focus is on the inaction of the central bank, not the reason for its inaction nor the level at which the policy rate is pegged.[4] In any event, the qualitative effect of a four-period peg is anticipated. The peak decline in the 10 year yield is now 31 bp (compared to 23bp), while the peak decline in the term premium is 44 bp (compared to 36 bp). The response of investment (and thus output) to this decline in long rates is substantial, with investment peaking at over 13% above steady-state. As we increase the number of periods in which the policy rate is pegged, these effects become unboundedly large, and then reverse sign. In the present context, this reversal occurs at an interest rate peg of only six quarters. But this peculiar behavior is not caused by the segmentation model. Instead, and as emphasized by Carlstrom, Fuerst, and Paustian (2012), these reversals are endemic in the underlying DNK model.

Sensitivity analysis on the QE experiment is reported in Table 1. The first observation is that the quantitative results are relatively unaffected by the size of the adjustment costs on net worth, ψ_n. The peak investment response only varies from 8.1% to 10.4% as we vary the adjustment cost parameter from 0.25 to 2. Similarly, the degree of price stickiness has only modest effects on the response of investment and the long yield. Fundamentally, the "loan-in-advance" constraint is a real constraint, so that QE has an effect even in a flexible price world. The remaining parameters in Table 1 have a bigger quantitative impact. From (41), higher levels of

[4] This approach to the ZLB follows Carlstrom, Fuerst, and Paustian (forthcoming), in their analysis of fiscal multipliers.

steady state leverage dampen the effect of movements in leverage on the loan-deposit spread, so that QE shocks have a smaller effect. As for balance sheet effects, if FIs hold relatively little government debt, then purchases of debt have a smaller effect on their balance sheet and thus a smaller effect on real activity. But this is a statement about percentage changes. The effect of an absolute change in the FI's holdings of government debt is invariant to the steady state holdings of government debt. Hence, the model implies that there are no "diminishing returns" to a given absolute purchase of government debt. But these absolute QE effects are inversely related to the steady state net worth position of FIs.

Finally, Table 1 demonstrates the quantitative importance of the duration of the investment bonds. These different durations imply significantly different steady state distortions as M_{ss} is increasing in the duration of the investment bond. Figure 3 plots this relationship, with M_{ss} varying from 1.04 at 20 quarters, 1.07 at 40 quarters, and 1.11 at 80 quarters. This steady state effect is also manifested in the dynamic effect of a QE shock as longer maturities for the investment bonds lead to much larger effects on both real activity and the long bond term premium. For example, as we move from 5 year bonds to 20 year bonds, the peak investment response increases from 7.2% to 12.5%, and the long rate response increases from 20bp to 29bp. Recall that without a borrowing constraint for investment, changes in the term premium would have no real effects.[5]

b. **<u>Other shocks under exogenous and endogenous debt policies.</u>**

Figures 4-7 look at the effect of an MEI shock, a TFP shock, a credit shock, and a monetary shock, respectively, under a policy that holds the long bonds in the balance sheet fixed ("exogenous debt"), or holds the term premium fixed ("endogenous debt"). As noted earlier, the fixed term premium policy largely stabilizes the market segmentation distortion so that the IRFs to these shocks will closely resemble their DNK counterparts. The real shocks are both set to 1%, and the monetary shock is 100 annual bp. The TFP shock has an

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[5] If we vary the duration of government debt, κ, with an exogenous debt policy there will be no real impact. Government bond duration separates out from the rest of the system (see equations 39 and 40) so κ only affects bond pricing and the term premium.

autocorrelation of 0.95, the MEI and credit shocks have autocorrelation equal to 0.8, and the monetary shock has autocorrelation of 0.6.

In response to the MEI shock, the term premium moves only modestly (peaks at less than 2 bp) under the exogenous debt scenario. This then implies that the IRFs are similar between the two debt polices. But the modest differences are suggestive. Note that under the exogenous debt policy, the term premium rises (a greater distortion) but the inflation increase implies that marginal cost also rises (a smaller distortion). Hence, the exogenous debt policy seems advantageous as the two distortions co-vary negatively. But of course these effects are small as the term premium movement is small. We will see these (small) effects manifested in the welfare analysis below.

In contrast to the investment shock, the TFP shock leads to a significant increase in the term premium (nearly 20 bp) under an exogenous debt policy, so that investment declines and thus dampens the response of output to the shock. This increase in the term premium is associated with a 59 bp decline in the short rate induced by the deflationary pressure of the TFP increase. The deflation increases the real value of investment debt, and coupled with the inertial movement in the FI's balance sheet, leads to a significant increase in the term premium. From a welfare point of view, the exogenous debt policy seems quite problematic. The increased marginal cost distortion (decline in inflation) is coupled with an increase in the market segmentation distortion (increase in the term premium), ie., the distortions co-vary positively. In contrast, the term premium peg (endogenous debt) eliminates movements in the segmentation distortion and dampens the change in the marginal cost distortion. This policy is supported by an endogenous QE policy that peaks at nearly 3%. In summary, in response to a TFP shock, the term premium peg is likely welfare-enhancing, and these effects are likely to be large (in comparison to the MEI shocks).

The welfare advantage of the term premium peg is particularly obvious for the case of credit shocks. Under a term premium peg, the central bank entirely sterilizes these shocks by engaging in an aggressive bond purchase program. Hence, the credit shocks have no real effects. But under the exogenous debt policy, the credit shock leads to a significant increase in the long rate (32 bp) and term premium (36 bp), with corresponding real effects. As with a QE shock, the term premium returns to steady state before the long rate because the overshooting of the short rate eventually dominates the determination of the long rate. Finally, the monetary shock induces a significant increase in the term premium which amplifies the effect of the monetary shock (compared to its DNK counterpart).

Finally, Figure 7 looks at the case of a 100 bp monetary shock. One important observation is that the exogenous debt policy magnifies the effect of the monetary shock by sharply increasing the term premium. This then explains the nearly doubling of the real effects on output, investment and consumption.

c. <u>**Welfare consequences of a Taylor rule including the term premium.**</u>

In this section we consider the effect of a central bank including the term premium in its FFR Taylor rule. In particular, suppose that the Taylor Rule is given by:

$$r_t = \rho r_{t-1} + (1-\rho)(\tau_\pi \pi_t + \tau_y y_t^{gap} + \tau_{tp} tp_t)$$

where the term premium (tp_t) is defined as in (56). As an initial experiment, we set the remainder of the Taylor rule at the benchmark parameter values ($\rho = 0.8$, $\tau_\pi = 1.5$, $\tau_y = 0.5$) and consider the welfare consequences of alternative values for τ_{tp}.

The first step in the analysis is to ensure equilibrium determinacy. Figure 8 looks at equilibrium determinacy for the Taylor rule that includes the term premium. For determinacy under a term premium rule, you cannot have too large of a response to the term premium. At the baseline calibration for the Taylor rule, this restriction is $\tau_{tp} < 0.76$. The reason is that responding positively to the term premium implies a negative response to the future path of the funds rate. But a negative response to the term premium is typically consistent with determinacy. Further, Figure 8 implies that as long as this response is not too negative, then the response to inflation can be significantly below unity.

Figures 9-11 look at the welfare consequence of alternative τ_{tp} for the baseline parameter calibration. We consider two real shocks: TFP shocks, and MEI shocks. We choose the standard deviation of the shocks so that the standard deviation of output equals 0.02 under the baseline parameter calibration. Figure 9 focuses only on MEI shocks and sets the MEI SD = 0.033. Figure 10 does the complementary exercise for the TFP shocks and sets the TFP SD = 0.015. Figure 11 looks at both shocks. In this case we set the relative SDs so that the 80% of the variability of investment comes from the MEI shock. This is consistent with the evidence in Justiniano, Primiceri, and Tambalotti (2010, 2011). This calibration implies SD of MEI = 0.027, and SD of TFP = 0.009. The units are in consumption perpetuities, i.e., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%.

Figure 9 considers the case of MEI shocks. Although the preferred response is positive, there is only a modest gain to having the central bank respond to the term premium. This result is anticipated by Figure 4 in that the IRFs under a policy with a term premium peg is not that different than under a Taylor rule with zero response to the premium. The efficacy of a positive response to the term premium evidently comes from the desire to dampen movements in inflation. This small but positive welfare gain is somewhat seductive. As we increase the term premium response, we start approaching indeterminacy and the existence of welfare-reducing sunspot equilibria.

With TFP shocks in Figure 10, the welfare gain of a term premium response is significant (roughly 20 times the magnitude of MEI shock). The optimal response is negative, so that indeterminacy of equilibrium is not an issue. The explanation comes from Figure 5. With no reaction to the term premium, the TFP shock leads to a significant increase in the segmentation distortion (as the term premium rises sharply), coupled with a significant increase in the marginal cost distortion (as inflation falls sharply). But figure 7 suggests that the central bank can ameliorate these effects by lowering the policy rate in response to the increased term premium. This dampens the movement in inflation and the marginal cost distortion. The welfare gains can be significant, in excess of 0.3% consumption perpetuity.

Figure 11 considers both shocks, calibrated to match a SD of 2%, with 80% of the variance of investment coming from MEI shocks. The results are as anticipated: there is a modest gain to responding negatively to increases in the term premium. The welfare gain is on the order of a 0.1% consumption perpetuity.

We have conducted a similar analysis for the case of flexible prices, $\theta_p = 0$. The results are qualitatively and quantitatively similar. This suggests that the welfare gain of responding to the term premium is independent of the degree of price stickiness in the model. Finally, Table 2 considers two stark policies. In both cases, the central bank uses the baseline Taylor rule (without a response to the term premium). In terms of long debt policy, we consider two extremes: (i) the level of long debt in circulation is held fixed (so that the term premium is endogenous), vs. (ii) the term premium is pegged (so that the level of long debt is endogenous). Note that the term premium peg does not completely stabilize the market segmentation distortion m_t, but its variability is lowered by an order of magnitude. As the previous results suggest, there are welfare gains to stabilization of the term premium, although these gains become more modest if MEI shocks are the principle source of the business cycle. Table 2 also confirms the hunch that the welfare gains are largely independent of the degree of price stickiness.

d. **Indexed debt.**

An important assumption in the model sketched above is that the bonds are nominal payments, i.e., κ and κ_I, are nominal. With nominal bonds positive innovations in inflation lead to negative innovations in the real value of investment bonds. For given levels of net worth and government debt, this frees up the FI balance sheet for the acquisition of new investment debt, and thus makes investment finance cheaper (see expression (37)). For example, with non-indexed debt, the QE shock in Figure 2 leads to an increase in inflation that amplifies the effect of the QE shock by lowering the real value of existing investment debt. We have investigated the model under the assumption that the debt payments are indexed, i.e., κ and κ_I, are real. For the QE shock, the effects on the term premium and real activity are dampened but still significant (the peak change in the long rate is 21 bp, while the investment peak response is now 8.5%). Similarly, there is only a trivial effect of indexation on the MEI shock because the inflation movement is so modest.

The two most significant changes from indexed debt are in the IRFs to the TFP shock and monetary policy shock. With non-indexed debt, the TFP shock leads to a decline in investment because of the increase in the real value of existing investment debt. This balance sheet effect is entirely absent with indexed debt, so that the term premium increases only modestly with a TFP shock, and investment rises sharply. This change in investment behavior also implies that the welfare gains to term-premium targeting decline by an order of magnitude.

As for the monetary policy shock and non-indexed debt, the monetary shock has two contrasting effects: (i) an increase in the term premium because of the increase in the real value of existing investment debt, and (ii) a decrease in the term premium because of the reduced demand for investment. Quantitatively the first effect wins so that the monetary shock increases the term premium. But for indexed debt, the first effect is absent, and the lower demand for investment leads to a decline in the term premium. Hence, the response of the term premium to monetary policy shocks depends fundamentally on whether or not the existing investment debt is indexed.

4. Conclusion.

This paper has built a model to analyze the Quantitative Easing policy used by the Fed during the recent ZLB environment. At the core of any such model is an assumption about market segmentation. In the present model we assume the short term money market is segmented from the long term bond market. Households buy long-term debt instruments indirectly by providing deposits to intermediaries. But intermediaries are limited in their ability to arbitrage the return differentials because the amount of deposits is constrained by an intermediary's net worth. Risk neutral intermediaries would immediately increase net worth to eliminate these movements but the intermediary faces adjustment costs in varying the size of its portfolio. Finally these long-term yields affect real economic activity because of a loan-in-advance constraint for capital investment.

We show that the real impact of this segmentation is meaningful. These real effects arise because the assumed segmentation introduces a time-varying wedge or distortion on the cost of investment goods. But any wedge needs a remedy. We emphasize two results. First, a monetary policy that targets the term premium in a Taylor rule can largely eliminate movements in the distortion from market segmentation. In particular, welfare is improved modestly when the short-term rate responds negatively to the term premium. Second, a policy that makes the balance sheet endogenous by directly targeting the term premium will sterilize credit shocks. The advantage of this sterilization depends quite naturally on the importance of credit shocks in the business cycle.

We have assumed that government and private sector bonds are perfect substitutes. Hence, when government bonds are purchased from intermediaries, they respond by replacing public with private debt one for one. In practice, this link is less strong because of imperfect substitutability. Hence, our model is likely to give an upper bound on the impact of asset purchases. It would be useful to extend the model to include imperfect substitutability. We leave this to future work.

APPENDIX.

A. Nonlinear equilibrium conditions:

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - E_t \frac{\beta h}{C_{t+1} - hC_t} \tag{A1}$$

$$\Lambda_t = E_t \beta \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \tag{A2}$$

$$BL_t^{\eta} = W_t \Lambda_t \tag{A3}$$

$$\Lambda_t P_t^k M_t = E_t \beta \Lambda_{t+1} [R_{t+1}^k + (1-\delta) P_{t+1}^k M_{t+1}] \tag{A4}$$

$$\Lambda_t Q_t^I M_t = E_t \beta \Lambda_{t+1} \frac{[1 + \kappa_I Q_{t+1}^I M_{t+1}]}{\Pi_{t+1}} \tag{A5}$$

$$R_t^k = MC_t MPK_t \tag{A6}$$

$$W_t = MC_t MPL_t \tag{A7}$$

$$\Pi_t^* = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1t}}{X_{2t}} \Pi_t \tag{A8}$$

$$X_{1t} = MC_t \Lambda_t Y_t + \beta \theta_p \Pi_t^{-\iota_p \epsilon_p} \Pi_{t+1}^{\epsilon_p} X_{1t+1} \tag{A9}$$

$$X_{2t} = \Lambda_t Y_t + \beta \theta_p \Pi_t^{\iota_p (1-\epsilon_p)} \Pi_{t+1}^{\epsilon_p - 1} X_{2t+1} \tag{A10}$$

$$\Pi_t^{1-\epsilon_p} = \left(1 - \theta_p\right) (\Pi_t^*)^{1-\epsilon_p} + \theta_p \Pi_{t-1}^{\iota_p (1-\epsilon_p)} \tag{A11}$$

$$d_t = \Pi_t^{\epsilon_p} \left[\left(1 - \theta_p\right) (\Pi_t^*)^{-\epsilon_p} + \theta_p \Pi_{t-1}^{-\iota_p \epsilon_p} d_{t-1}\right] \tag{A12}$$

$$C_t + I_t = Y_t \tag{A13}$$

$$Y_t = \frac{A_t K_t^{\alpha} L_t^{1-\alpha}}{d_t} \tag{A14}$$

$$K_t = (1-\delta) K_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{A15}$$

$$P_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right)\right] = 1 - \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1}^k \mu_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right) \tag{A16}$$

$$\bar{B}_t + \bar{F}_t \leq N_t \left[1 - \Phi_t \left(\frac{R_t^L}{R_t}\right)\right]^{-1} \tag{A17}$$

$$P_t^k I_t \leq \bar{F}_t - \kappa_I \frac{\bar{F}_{t-1}}{\Pi_t} \frac{Q_t^I}{Q_{t-1}^I} \tag{A18}$$

$$\Lambda_t[1 + N_t f'(N_t) + f(N_t)] = E_t \beta \zeta \Lambda_{t+1} \frac{R_t^L}{\Pi_{t+1}} \tag{A19}$$

$$R_t^L = E_t \left(\frac{1 + \kappa_I Q_{t+1}^I}{Q_t^I} \right) \tag{A20}$$

$$R_t^L = E_t \left(\frac{1 + \kappa Q_{t+1}}{Q_t} \right) \tag{A21}$$

$$R_t^{10} = Q_t^{-1} + \kappa \tag{A22}$$

$$ln(\bar{B}_t) = \left(1 - \rho_1^b - \rho_2^b\right) ln(\bar{B}_{ss}) + \rho_1^b \, ln(\bar{B}_{t-1}) + \rho_2^b \, ln(\bar{B}_{t-2}) + \varepsilon_t^b \tag{A23}$$

$$ln(R_t) = (1 - \rho) \, ln(R_{ss}) + \rho \, ln(R_{t-1}) + (1 - \rho)\left(\tau_\pi \pi_t + \tau_y y_t\right) + \varepsilon_t^r \tag{A24}$$

$$ln(A_t) = \rho_a \, ln(A_{t-1}) + \varepsilon_t^a \tag{A25}$$

$$ln(\Phi_t) = (1 - \rho_\phi) \, ln(\Phi_{ss}) + \rho_\phi \, ln(\Phi_{t-1}) + \varepsilon_t^\phi \tag{A26}$$

$$ln(\mu_t) = \rho_\mu \, ln(\mu_{t-1}) + \varepsilon_t^\mu \tag{A26}$$

where

$$f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}} \right)^2$$

$$S\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\psi_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2$$

$$\bar{B}_t \equiv Q_t \frac{B_t}{P_t}$$

$$\bar{F}_t \equiv Q_t^I \frac{F_t}{P_t}$$

B. Steady State:.

We choose B so that $L_{ss} = 1$. We also normalize $\mu_{ss} = A_{ss} = 1$.

$$\Lambda_{ss} = \frac{(1-\beta h)}{(1-h)C_{ss}}$$

$$1 = \beta R_{ss}$$

$$B = W_{ss}\Lambda_{ss}$$

$$R_{ss}^{k} = \frac{M_{ss}[1-\beta(1-\delta)]}{\beta}$$

$$M_{ss} = \frac{\beta}{(1-\beta\kappa_I)Q_{ss}^{I}}$$

$$R_{ss}^{k} = MC_{ss}MPK_{ss}$$

$$W_{ss} = MC_{ss}MPL_{ss}$$

$$1 = \frac{\epsilon_p}{\epsilon_p - 1}\frac{X_{1ss}}{X_{2ss}}$$

$$X_{1ss} = \frac{MC_{ss}\Lambda_{ss}Y_{ss}}{1-\beta\theta_p}$$

$$X_{2ss} = \frac{\Lambda_{ss}Y_{ss}}{1-\beta\theta_p}$$

$$d_{ss} = 1$$

$$C_{ss} + I_{ss} = Y_{ss}$$

$$Y_{ss} = K_{ss}^{\alpha}$$

$$\delta K_{ss} = I_{ss}$$

$$P_{ss}^{k} = 1$$

$$\bar{B}_{ss} + \bar{F}_{ss} = N_{ss}\left[1 - \Phi_{ss}\left(\frac{R_{ss}^{L}}{R_{ss}}\right)\right]^{-1}$$

$$I_{ss} = \bar{F}_{ss}(1-\kappa_I)$$

$$1 = \beta\zeta R_{ss}^{L}$$

$Q = (R_{ss}^{L} - \kappa)^{-1}$

$Q_I = (R_{ss}^{L} - \kappa_I)^{-1}$

$R_{ss}^{10} = R_{ss}^{L}$

<u>Some simplifications:</u>

$$M_{ss} = \frac{(\beta R_{ss}^{L} - \beta\kappa_I)}{(1 - \beta\kappa_I)} > 1$$

$$MC_{ss} = \frac{\epsilon_p - 1}{\epsilon_p} < 1$$

$$\frac{K_{ss}}{Y_{ss}} = \frac{\beta\alpha MC_{ss}}{M_{ss}[1 - \beta(1 - \delta)]}$$

$$K_{ss} = \left(\frac{K_{ss}}{Y_{ss}}\right)^{1/(1-\alpha)}$$

$$C_{ss} = Y_{ss} - \delta K_{ss}$$

C. <u>Calibration:</u>

$\beta = 0.99$, $h = 0.8$, $\eta = 0.25$, $\iota_p = 1$, $\epsilon_p = 5$, $\theta_p = 0.85$, $\psi_i = 2$, $\psi_n = 1$.

$\beta R_{ss}^{L} = 1.0025$

$$\Phi_{ss}(1.0025) = \frac{L_{ss} - 1}{L_{ss}}$$

$L_{ss} = 6$

duration = 40 = $(1 - \kappa_I)^{-1}$

duration = 40 = $(1 - \kappa)^{-1}$

$$\frac{\bar{B}_{ss}}{\bar{B}_{ss} + \bar{F}_{ss}} = 40\%$$

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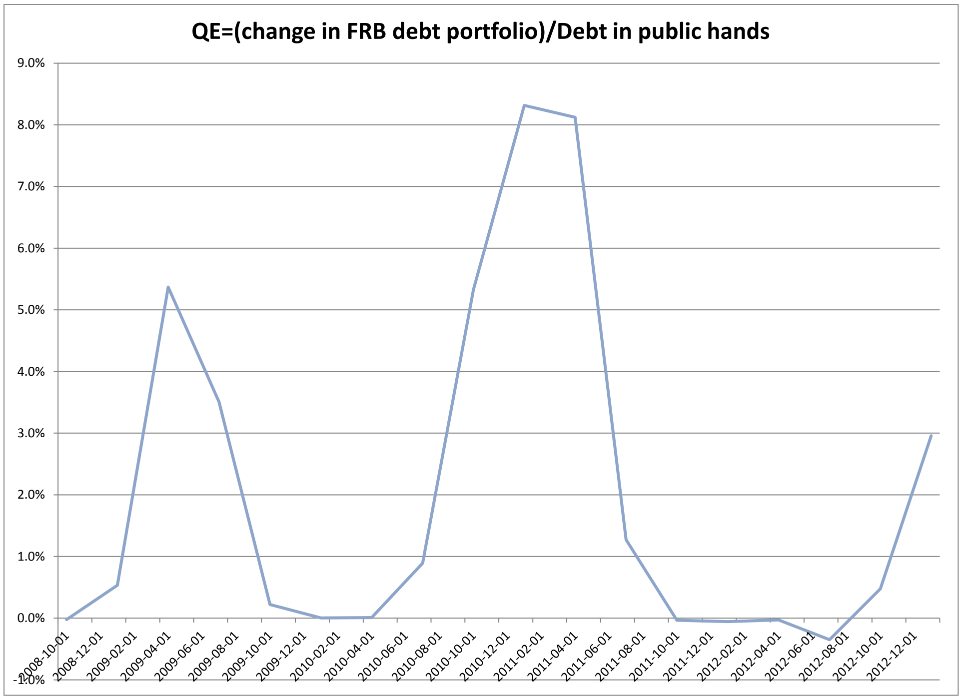
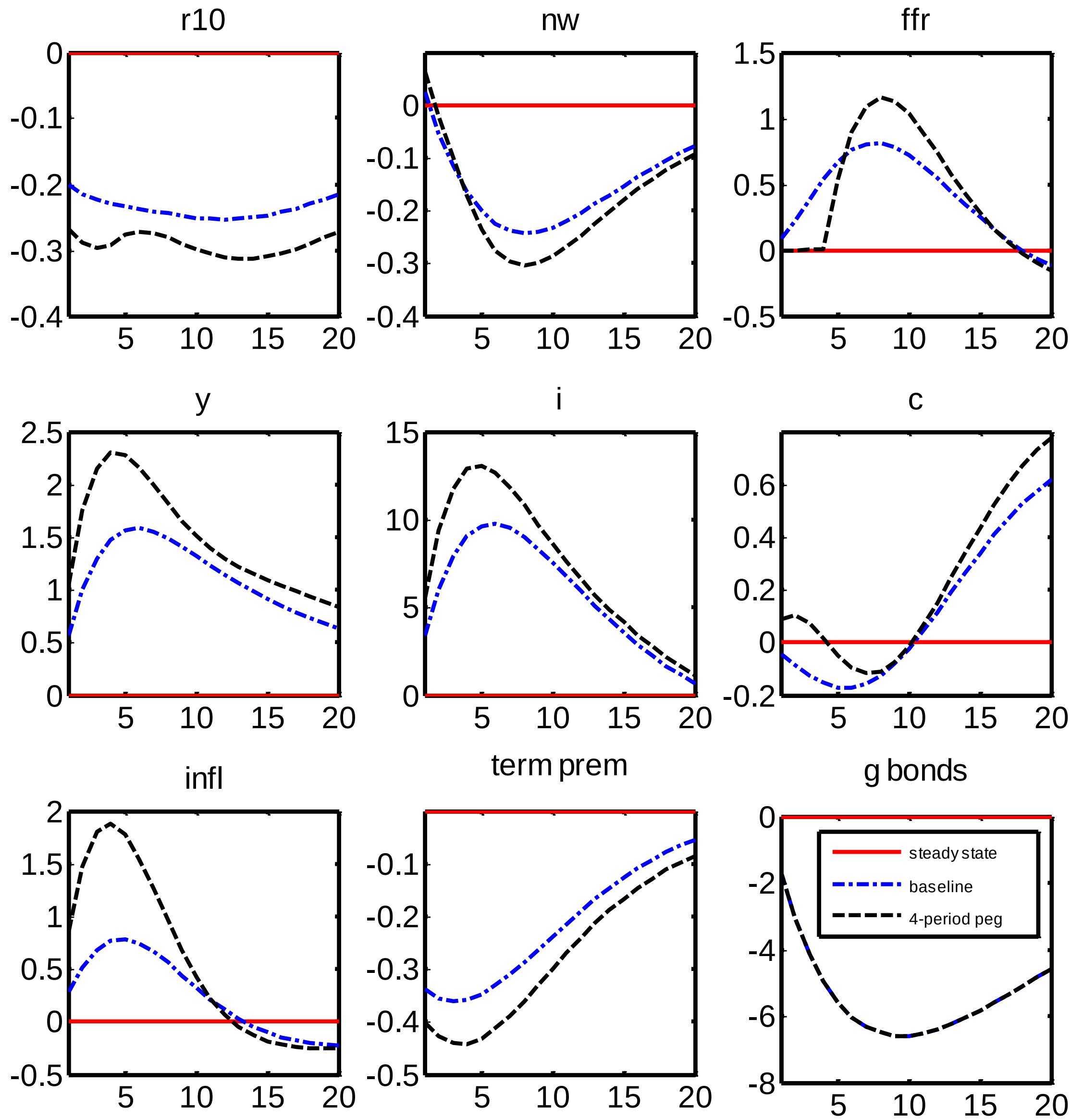
<u>Figure 1:</u>



Source: FRB St. Louis.

Figure 2: QE experiment, baseline parameter values.



Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 3: Effect of investment bond duration on M_{ss}.

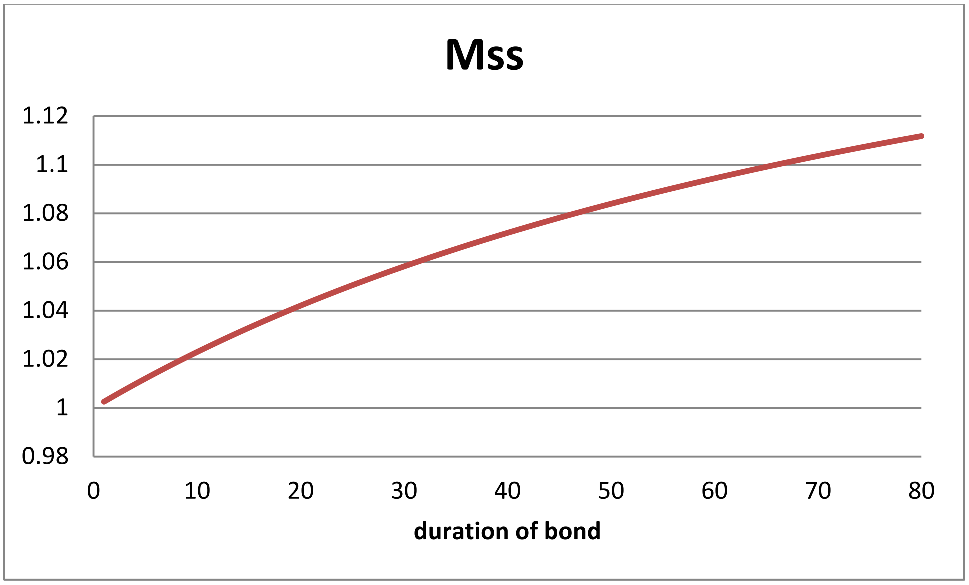
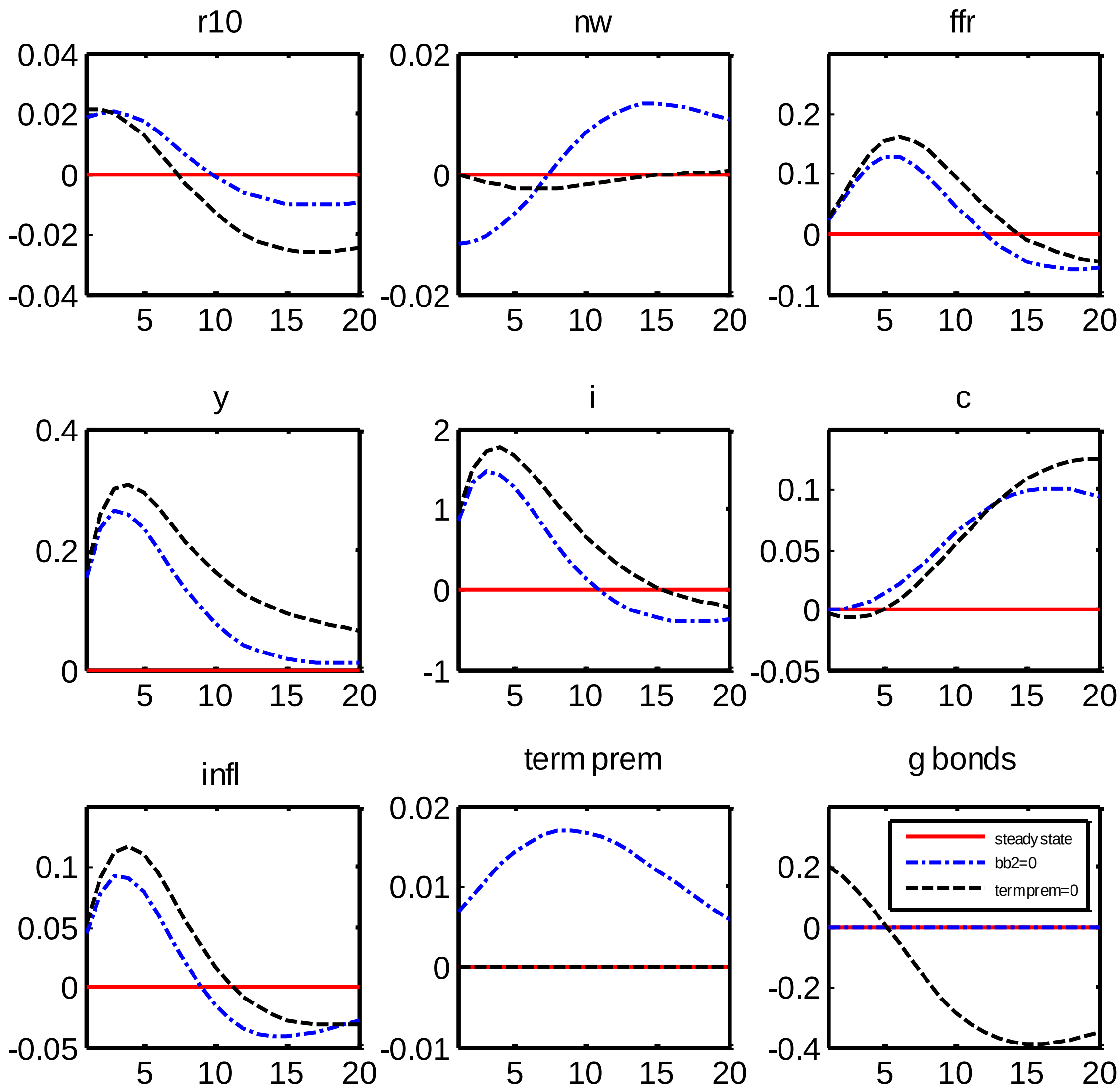
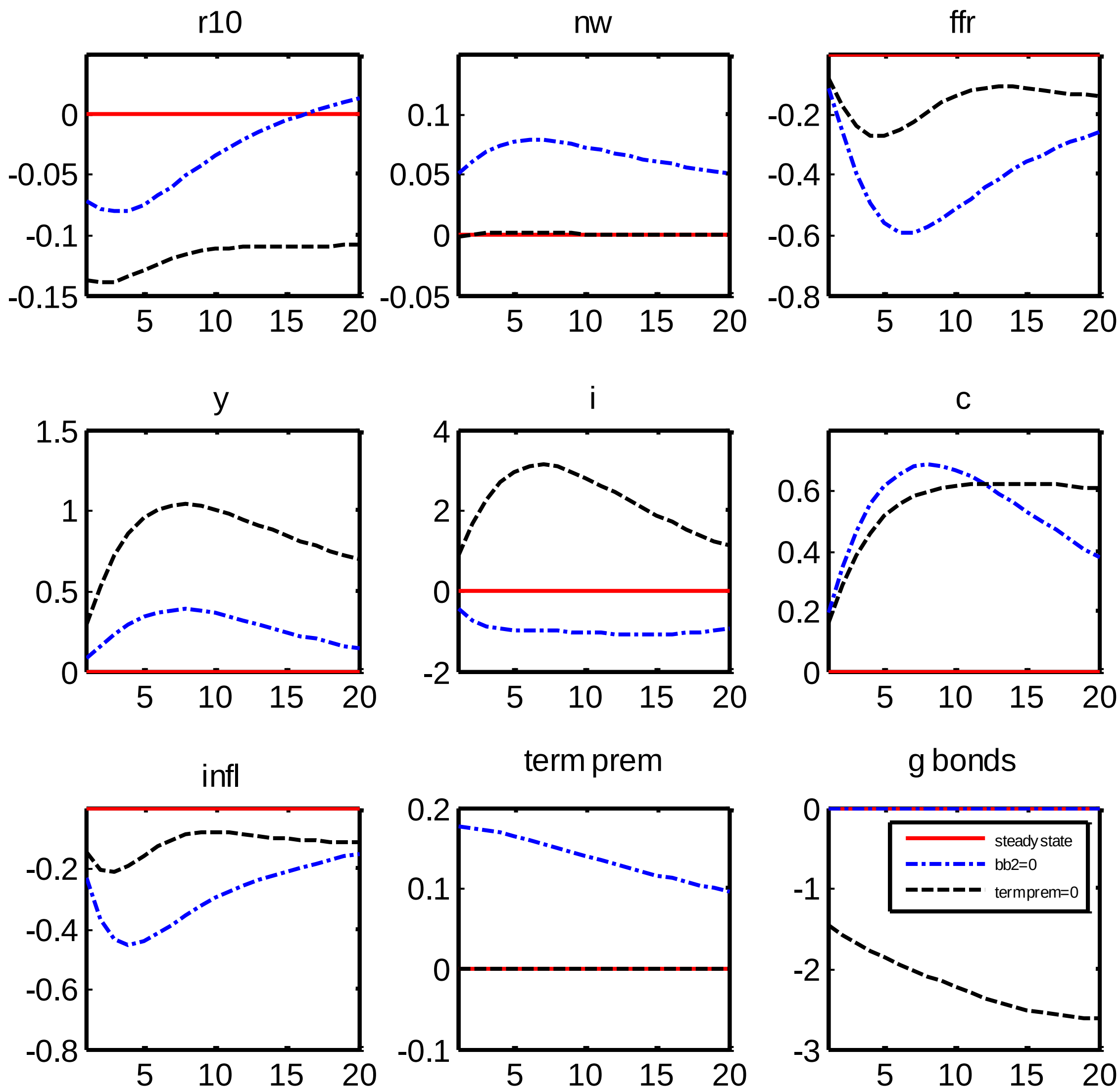


Figure 4: A 1% investment shock under exogenous and endogenous debt policies.



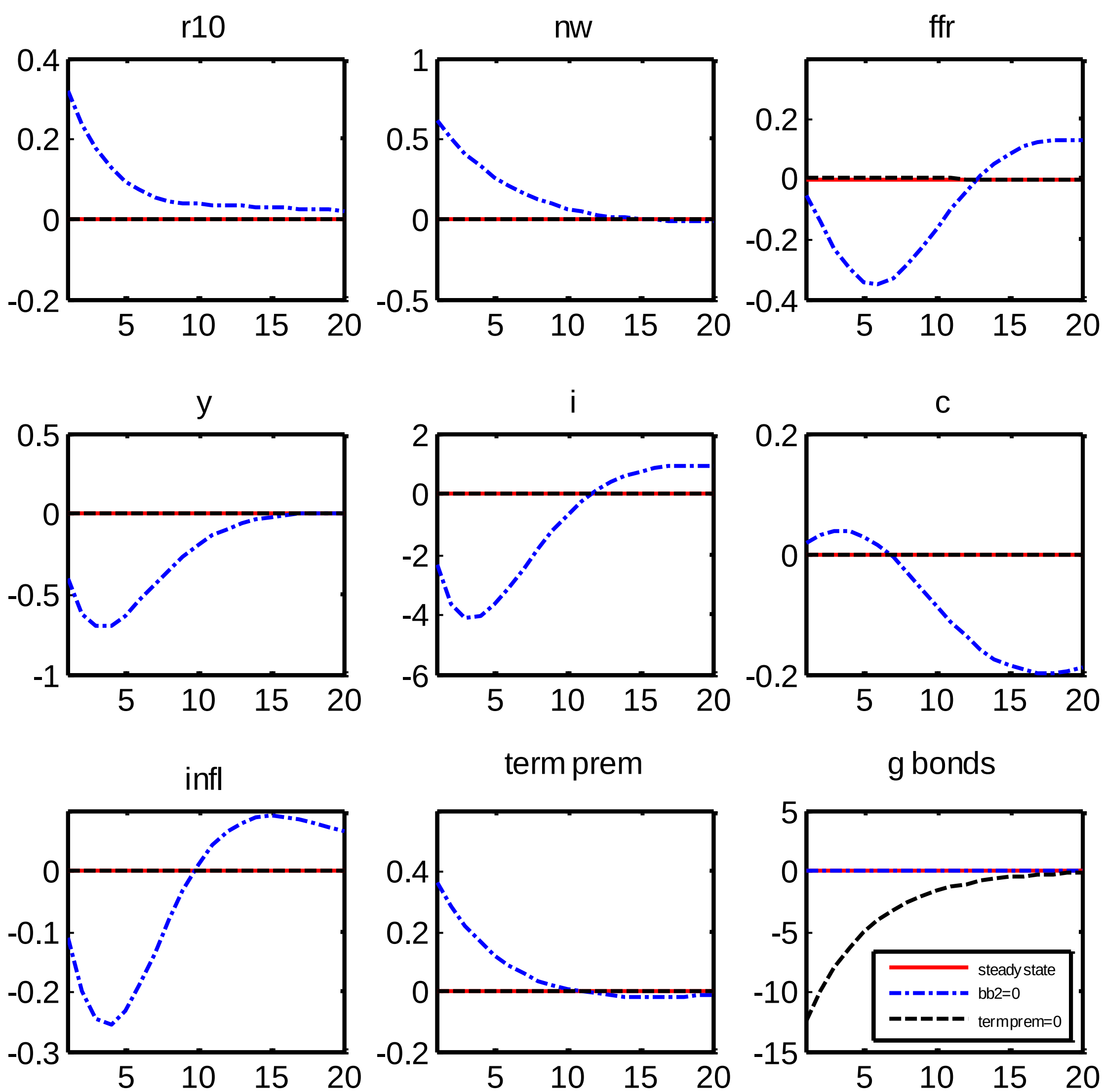
Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 5: A 1% TFP shock under exogenous and endogenous debt policies.



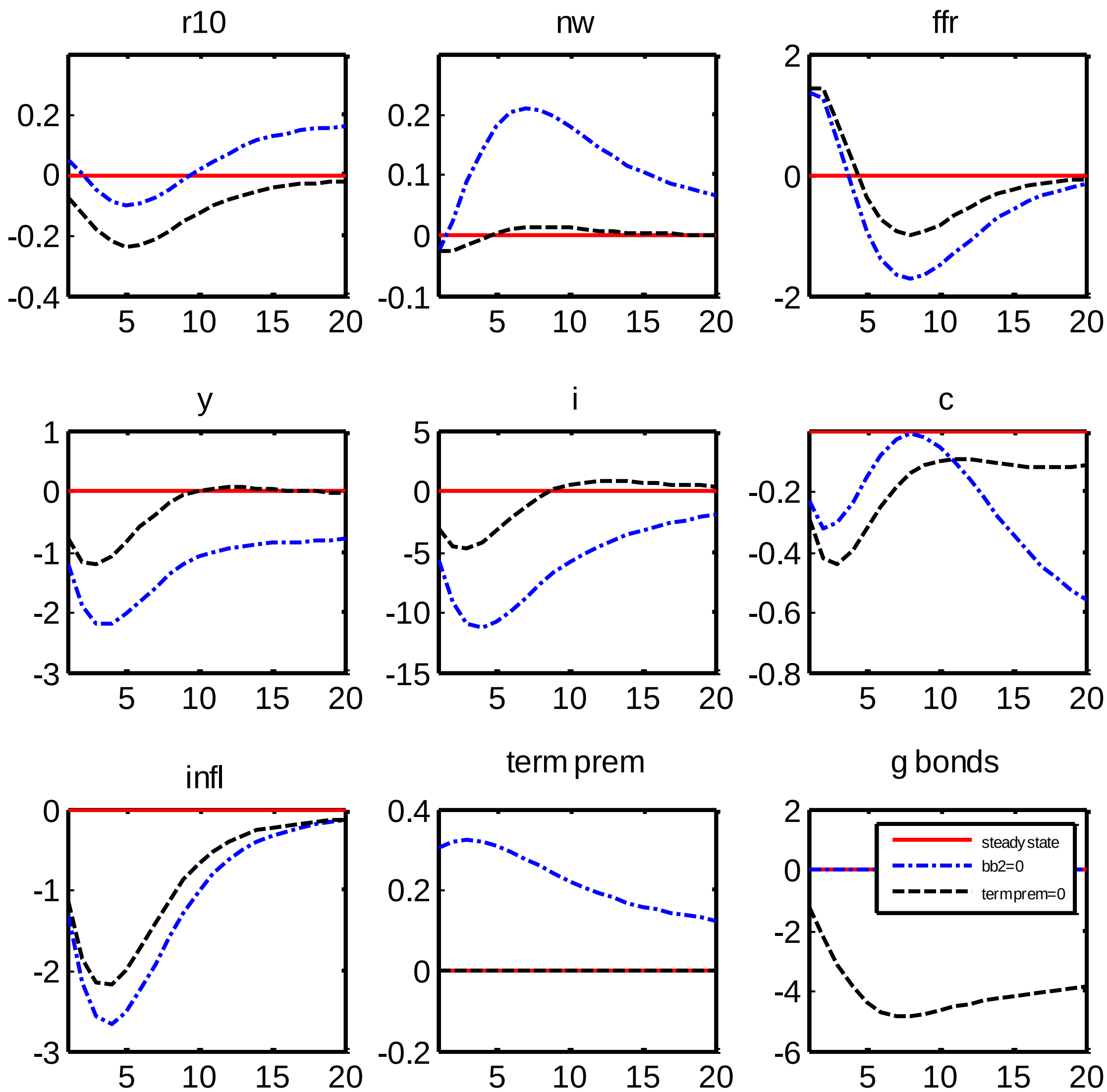
Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

<u>Figure 6: A 1% credit shock under exogenous and endogenous debt policies.</u>



Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 7: A 100 bp monetary shock under exogenous and endogenous debt policies.



Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 8: Equilibrium determinacy under a term premium Taylor Rule.

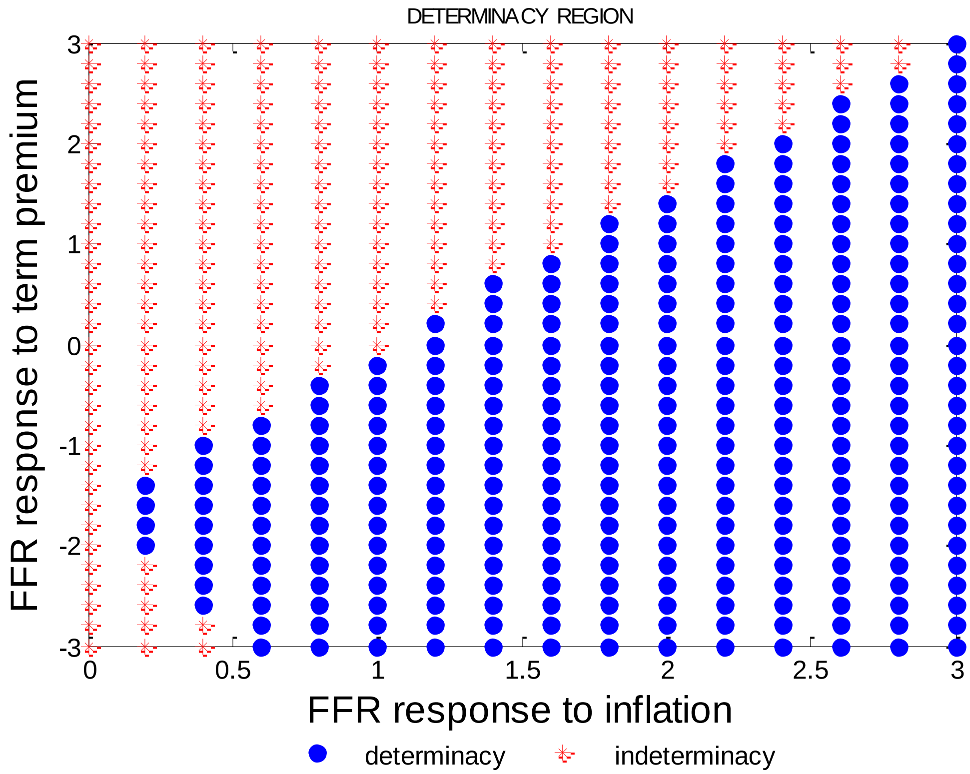
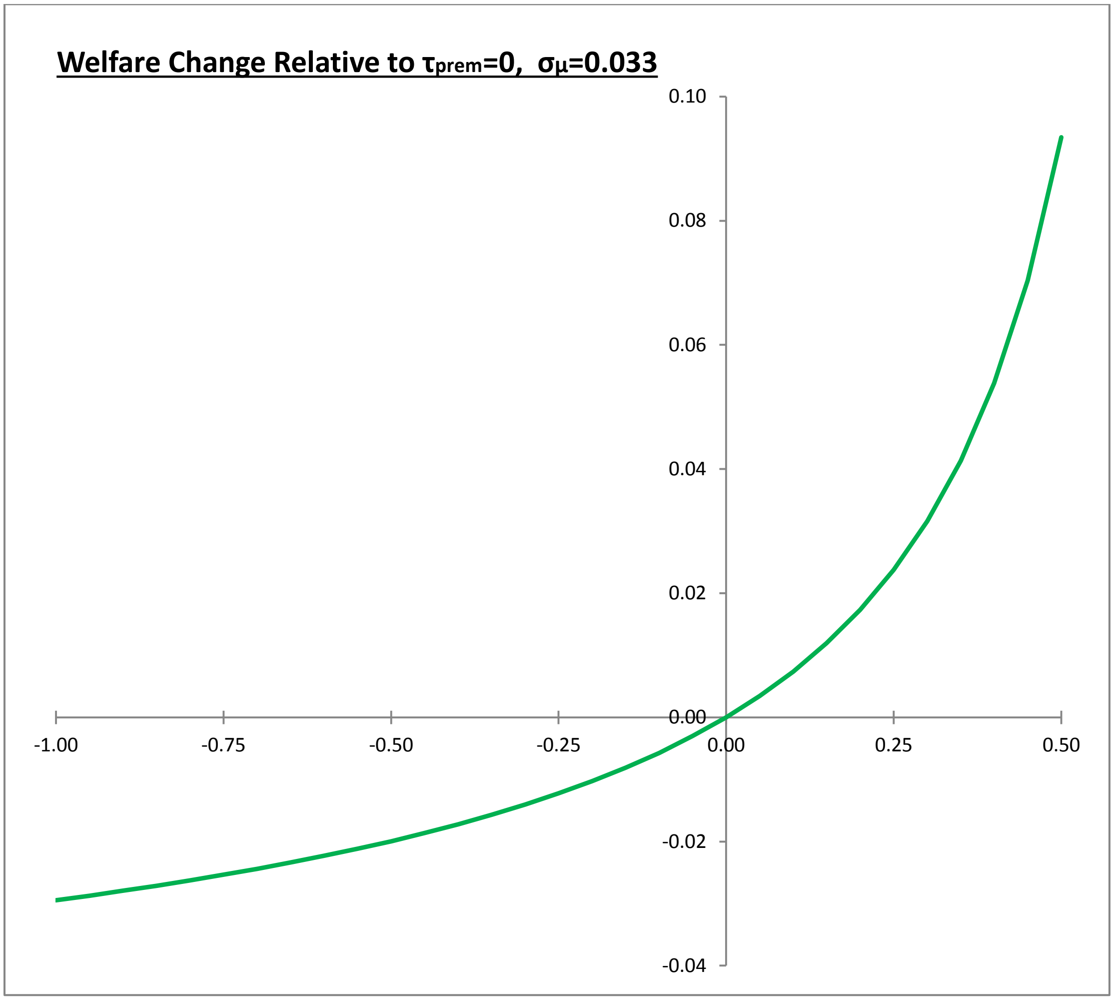
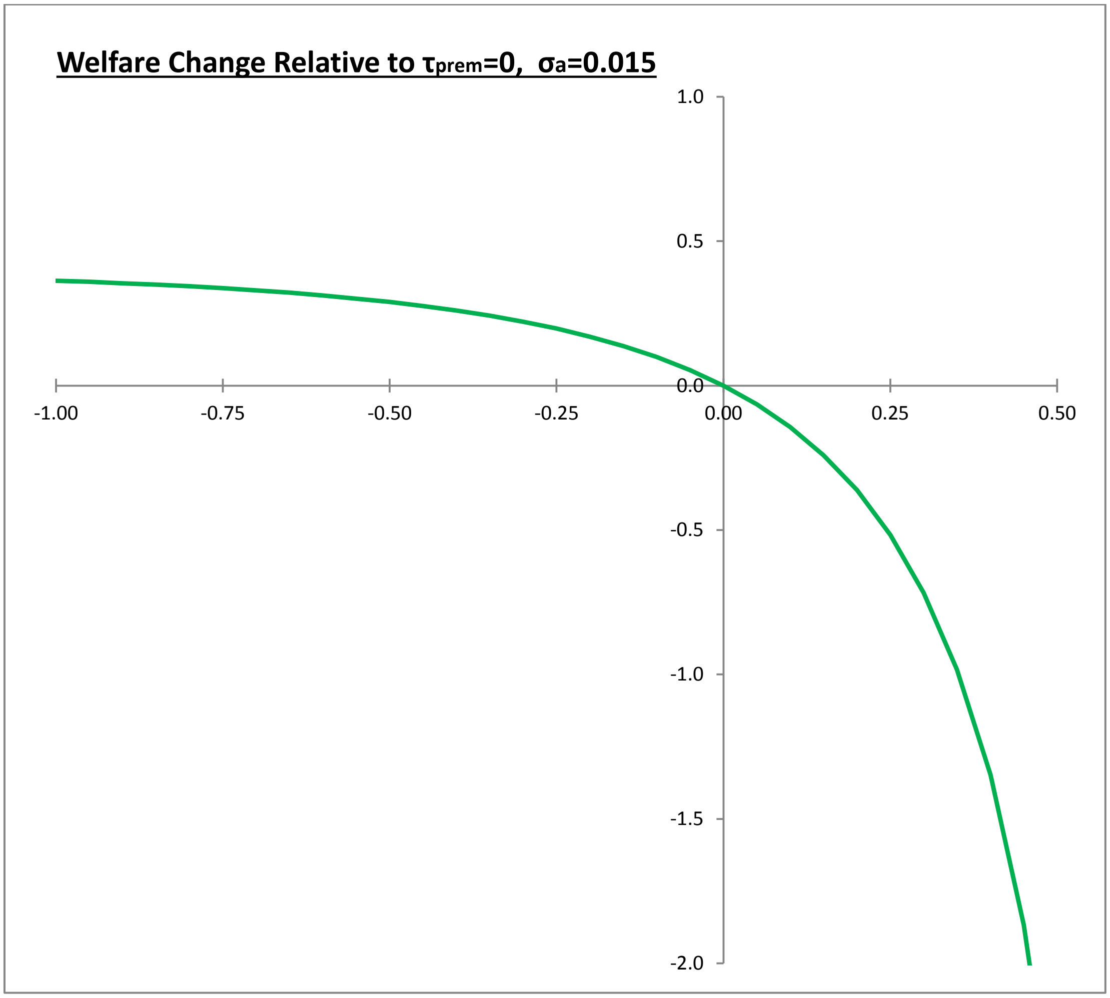


Figure 9: Welfare Consequences of Taylor Rule with term premium response. (Only MEI shocks)



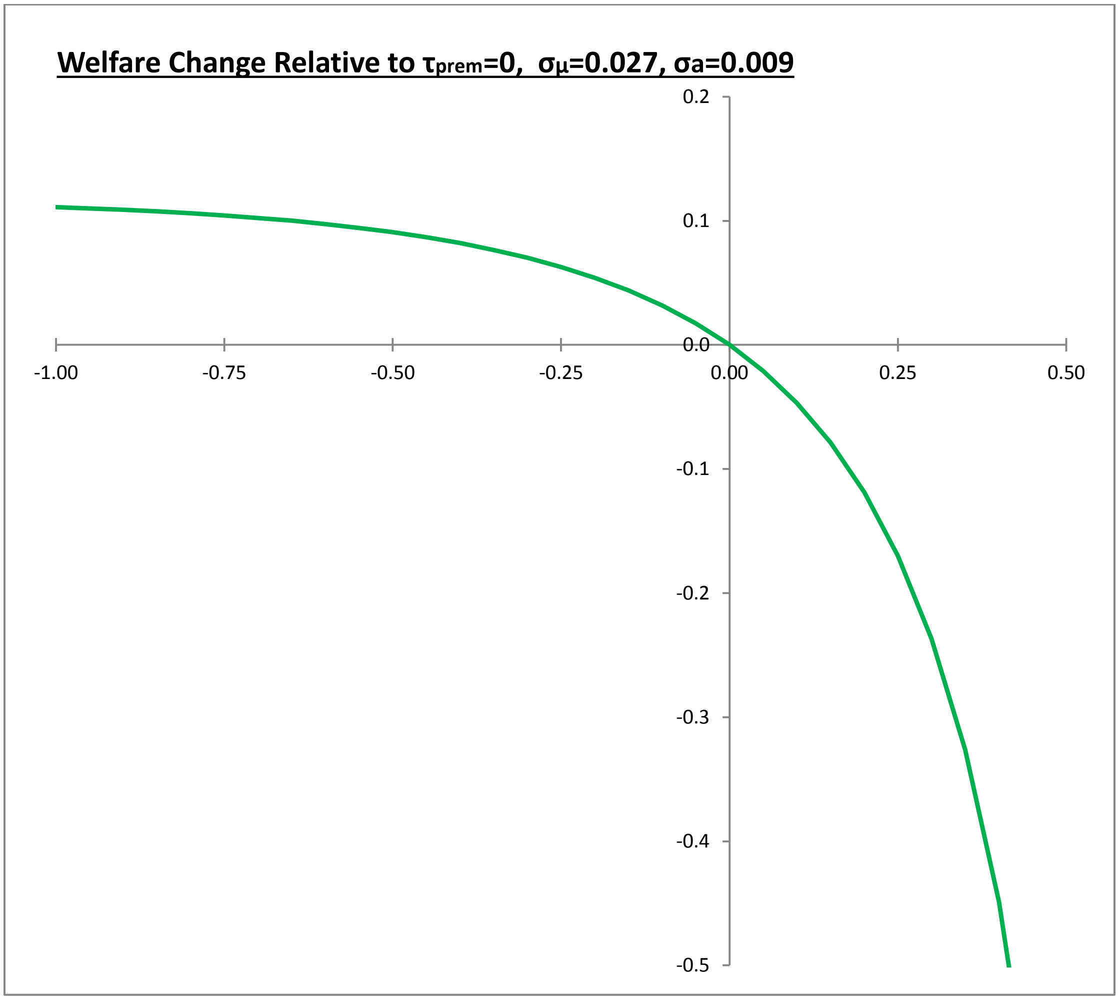
The units are in consumption perpetuities, ie., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%. The welfare change is on the vertical axis, and the term premium coefficient in the Taylor Rule is on the horizontal axis.

Figure 10: Welfare Consequences of Taylor Rule with term premium response.
(Only TFP shocks)



The units are in consumption perpetuities, ie., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%. The welfare change is on the vertical axis, and the term premium coefficient in the Taylor Rule is on the horizontal axis.

Figure 11: Welfare Consequences of Taylor Rule with term premium response. (TFP and MEI shocks)



The units are in consumption perpetuities, ie., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%. The welfare change is on the vertical axis, and the term premium coefficient in the Taylor Rule is on the horizontal axis.

Table 1: Sensitivity Analysis

This table contains sensitivity analysis of the effect of model parameters on the peak impact of a QE shock.

Parameter Value	Peak Investment Response	Peak Ten-Year Yield Response	Peak Inflation Response
Baseline*	**9.98**	**-0.23**	**0.69**
$\psi_n = 0$	0	0	0
$\psi_n = 0.25$	8.05	-0.19	0.54
$\psi_n = 0.5$	9.23	-0.21	0.63
$\psi_n = 2$	10.40	-0.24	0.7278
Dur_Inv = 20	7.17	-0.20	0.5049
Dur_Inv = 80	12.54	-0.29	0.8389
$\theta_p = 0$	9.70	-0.28	1.34
$\theta_p = 0.75$	9.80	-0.25	0.95
$\theta_p = 0.95$	9.93	-0.19	0.24
$\frac{Govt\ Debt}{Bank\ Assets} = 0$	0	0	0
$\frac{Govt\ Debt}{Bank\ Assets} = 0.2$	4.27	-0.10	0.30
$\frac{Govt\ Debt}{Bank\ Assets} = 0.8$	31.18	-0.78	2.01
Leverage = 3	13.37	-0.30	0.98
Leverage = 9	8.05	-0.19	0.54

*The baseline parameter values are $\psi_n = 1$, Dur_Inv = 40, $\theta_p = 0.85$, $\frac{Govt\ Debt}{Assets} = 0.4$, leverage = 6. All variables are expressed in percent, and rates are annualized.

Table 2: Comparing two stark policies.

Here we consider two start policy choices: holding the balance sheet fixed ($b_t = 0$), vs. a term premium peg, ($tp_t = 0$). The calibration of the SD of the shocks is as in Figures 10-12.

Sticky prices ($\boldsymbol{\theta_p = 0.85}$)			
	Both shocks	TFP alone	MEI alone
Welfare gain of term premium peg*	0.144	0.479	-0.042
SD of m with exogenous debt	4.8%	7.8%	1.4%
SD of m with term premium peg	0.3%	0.5%	0.2%

Flexible prices ($\boldsymbol{\theta_p = 0}$)			
	Both shocks	TFP alone	MEI alone
Welfare gain of term premium peg*	0.164	0.494	-0.020
SD of m with exogenous debt	5.1%	8.4%	1.3%
SD of m with term premium peg	0.3%	0.5%	0.2%

*The welfare units are in consumption perpetuities, i.e., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%.